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The effect of calculus and kinematics contexts on students' understanding of graphs

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Abstract
In this study, we compare students' understanding of the derivative as slope and the antiderivative as the area under the curve using isomorphic graph-based problems in both calculus and kinematics contexts. Drawing from previous research, we designed two isomorphic tests, each with 12 items, and administered them to 543 university students. Our findings show that students performed significantly better on the kinematics test, with a higher rate of correct answers in some items. We also identified the most frequent errors and general trends in students' selection of incorrect answers across both contexts. Based on these results, we provide specific recommendations for improving the instruction of these concepts. The analyses, recommendations, and tests included in the study can serve as valuable resources for mathematics and science education researchers and instructors teaching these topics. This study offers insights into how context influences students' conceptual understanding, with implications for enhancing calculus and physics education.

Keywords: students' graph understanding, concept of the derivative, concept of the antiderivative, educational innovation, STEM education

INTRODUCTION

In university-level introductory physics and mathematics courses, a key objective is for students to develop a strong understanding of graphs. In a mechanics course, this typically involves interpreting graphs related to kinematics, such as position, velocity, and acceleration. In contrast, a traditional calculus course in one dimension emphasizes understanding the essential features of a function's graph, particularly its first and second derivatives. Two fundamental concepts for grasping these graphical representations are the derivative as the slope of a curve and the antiderivative as the area under a curve. This article explores and compares students' comprehension of these concepts across the contexts of calculus and kinematics.

Numerous studies have examined students' understanding of slope and area under the curve in physics contexts (Beichner, 1994; Meltzer, 2004; Nguyen & Rebello, 2011; Woolnough, 2000). Similarly, other researchers have investigated this understanding within mathematical contexts (Christensen & Thompson, 2012; Hadjidemetriou & Williams, 2002; Leinhardt et al., 1990). Additionally, some studies have focused on comparing students' comprehension of these concepts across both physics and mathematics contexts (Carli et al., 2020; Ivanjek et al., 2016; Planinic et al., 2012, 2013; Susac et al., 2018).

We identified a gap in the literature for a comprehensive study comparing students' understanding of the relationships between variables across physics and mathematics contexts. Specifically, a study that evaluates all possible relationships between these variables and does so in an isomorphic manner, meaning the same types of questions are asked in both contexts. This research addresses that need. This comparison makes a detailed analysis of students' conceptual understanding possible. The significance of this study lies in its potential impact: the analyses, recommendations, and tests provided in **Appendix A**

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Contribution to the literature

- There is in the literature some studies comparing students' understanding of graphs in the contexts of mathematics and kinematics, focusing on students' understanding of the derivative concept as slope and the antiderivative concept as the area under the curve. However, we detected a need for a study that would conduct this comparison in a complete manner, that is, evaluating the understanding of all possible relationships between variables, and also in an isomorphic manner, that is asking about these variables in the same way in both contexts. This study covers this need. This comparison allows for a more detailed analysis of students' understanding.
- This study demonstrates that the context in which graph-related concepts of derivatives and antiderivatives are taught significantly impacts students' understanding and performance. That is students performed better in kinematics contexts than in the calculus context. The tangible nature of physical examples compared to the abstract nature of mathematical problems help students grasping the concepts better. This underscores the importance of context in educational settings and suggests that incorporating more real-world examples in calculus instruction could enhance understanding
- This study provides analyses, recommendations, and tests that can be valuable resources for mathematics and science education researchers, as well as for instructors teaching these concepts.

can serve as valuable resources for both mathematics and science education researchers, as well as instructors teaching these concepts.

In a previous paper (Zavala et al., 2017), we presented a modified version of the "test of understanding of graphs in kinematics (TUG-K)" designed by Beichner (1994). On the other hand, in another previous paper in this journal (Dominguez et al., 2017) we presented an isomorphic version of the TUG-K in the context of calculus ("test of understanding of graphs in calculus (TUG-C)." In those papers, we proved that both instruments are reliable tests with satisfactory discriminatory power. Using those tests, we designed two shorter versions of 12 items each that are used in the present investigation:

(1) a test with the context of calculus (referred to as "TUG-C short version") and

(2) a test with the context of kinematics (referred as "TUG-K short version").

These tests evaluate the derivative and antiderivative concepts from the relationship between a function's derivative and its second derivative.

The primary research question guiding this study is: "How do calculus and kinematics contexts influence engineering students' understanding of graphs, specifically the derivative as slope and the antiderivative as the area under the curve?" To address this question, we established three specific objectives:

(1) To analyze students' overall scores on both tests and assess the impact of context on these scores.

(2) To examine students' performance in each dimension of the two tests and evaluate the effect of context on this performance.

(3) To analyze students' performance on individual test items and determine how context influences their understanding.

It is worth noting that a preliminary analysis of this issue was presented in a brief article (Perez-Goytia et al., 2010), but the detailed analysis provided here expands significantly on that earlier work.

PREVIOUS RESEARCH

Studies That Analyze Students' Understanding of Slope and Area Under the Curve Concepts

Several studies investigate students' understanding of slope concepts in the context of kinematics (Beichner, 1994; Kusairi et al., 2020; McDermott et al., 1987; Planinic et al., 2012). Moreover, several studies investigate this understanding in the context of mathematics (Christensen & Thompson, 2012; Hadjidemetriou & Williams, 2002; Leinhardt et al., 1990; Planinic et al., 2012). Leinhardt et al. (1990) were the first to categorize students' difficulties into three groups:

(1) students tend to focus on a single point rather than an interval ("interval/ point confusion"),

(2) students tend to confuse the slope with the height of the graph ("slope/height confusion"), and

(3) students tend to consider the graph as a photograph of the situation ("iconic confusion").

Moreover, several studies analyze students' understanding of the area under the curve concept in the context of kinematics (Beichner, 1994; McDermott et al., 1987; Planinic et al., 2012). Also, several studies analyze this understanding in the context of mathematics (Bajracharya et al., 2012; Ballesteros et al., 2020; Martínez-Miraval & García-Cuéllar, 2020; Orton, 1983). Beichner (1994) categorized students' difficulties into three groups:

(1) not recognizing the meaning of area under the kinematics graph,

(2) calculating the slope rather than the area, and

(3) reading the value directly from the vertical axis.

Studies That Compare Students' Understanding in the Contexts of Physics and Mathematics

These studies are the most related to this article. Three articles compare this type (Carli et al., 2020; Ivanjek et al., 2016; Planinic et al., 2013). Here, we synthesize the most important findings presented in these articles.

The first two studies (Ivanjek et al., 2016; Planinic et al., 2013) were conducted by the same research group, with the second study building upon the first. In their initial study, Planinic et al. (2013) compared students' understanding of slope and area under the curve across three contexts: mathematics, kinematics, and a mixed context (including various scenarios such as price changes over time and population growth). The researchers used isomorphic multiple-choice and open-ended questions to assess students' understanding. Their findings showed that students found the mathematics context the easiest, while the other two contexts posed similar difficulty levels. For the concept of slope, student performance was relatively consistent across all three contexts. However, when it came to understanding the area under the curve, students performed better in the mathematics context, with comparable performance in the other two contexts.

Ivanjek et al. (2016) describe students' main strategies and difficulties identified through analyzing explanations and procedures in the second article. The authors establish the following main findings:

(1) the strategies used on parallel questions are often context-dependent and domain-specific,

(2) the preferred strategy on physics questions seems to be the use of formulas (often incorrect ones), and

(3) students show similar difficulties with graph interpretation in all domains.

In the third article, Carli et al. (2020) designed a multiple-choice test to compare students' ability to answer questions on derivatives, integrals, and vectors in two contexts: mathematics and physics. The researchers administered the test to university students finishing a calculus course in the first semester and starting physics in the second semester. Some of the main findings of the researchers are the following:

(1) the mean score was higher in the mathematics questions,

(2) students' attention was directed towards different distractors in a context-specific manner, and

(3) using formulas to calculate quantities in kinematics may limit students' problem-solving abilities.

At the end of this section, it is important to highlight the main differences between previous studies and our own. Two key distinctions exist between the studies by Planinic et al. (2013) and Ivanjek et al. (2016) and our research. The first difference lies in how questions were framed in the mathematics context. In the previous studies, students were asked directly to find the "slope" at a point or the "area" under a curve over an interval using a *y vs x graph*. In contrast, our study, in the context of calculus, asked students to find the derivative at a point or the change in the antiderivative over an interval. The second difference relates to the relationships evaluated in each context. In previous studies, the researchers assessed one relationship in the mathematics context and two relationships for each concept in the kinematics context. For the concept of "slope," they asked about "speed" (from the *x vs. t graph*) and "acceleration" (from the *v vs. t graph*). For the concept of "area," they asked about "distance traveled" (from the *v vs. t graph*) and "change in speed" (from the *a vs. t graph*). To build upon, our study evaluated both relationships across both contexts.

The third study has three key differences (Carli et al., 2020) and our research. The first difference relates to how questions about the antiderivative were framed. In their calculus test, students were asked to find the "definite integral," and in the kinematics test, they were asked to find the "displacement." In contrast, in our study, the calculus test asked students to determine the "the change of $f(x)$" from the graph of $f'(x)$, while the kinematics test asked them to find the "change in position" from the velocity graph. The second difference is that Carli et al. (2020) evaluated the concept of antiderivative in only one relationship, whereas our study evaluated it in both possible relationships. In the first relationship, we asked for the "change of $f(x)$" from the graph of $f'(x)$, and the "change in position" from the velocity graph. In the second relationship, we asked for the "change of $f'(x)$" from the graph of $f''(x)$, and the "change in velocity" from the acceleration graph. Our approach aligns with the concept of accumulated change, as recommended by Thompson and Silverman (2008), and with Jones' (2015) proposal of a multiplicatively based sum (Riemann sum), which is closely related to accumulated change.

The third difference is that Carli et al. (2020) evaluated the concept of the derivative in only one relationship. In contrast, our study also examined the second relationship, assessing students' understanding of the "second derivative" in calculus and "acceleration" in kinematics. It is important to highlight that our experimental design builds upon previous studies, aiming to provide a more comprehensive and isomorphic comparison between contexts. We will further explore how our findings relate to and expand upon the results of these earlier studies.

METHODOLOGY

The methodology of this study is divided into two subsections: participants and instruments. The participants' subsection provides details about the sample of engineering students who participated in the study. In contrast, the instruments subsection describes the tools used to assess their understanding of the derivative and antiderivative concepts across calculus and kinematics contexts.

Participants

The study was conducted at a large private university in Mexico, involving engineering students completing their introductory calculus-based mechanics course and their first calculus course. The mechanics course is the first of three introductory physics courses required for students at this institution. The calculus course covers key topics such as linear functions, qualitative analysis of functions and their first and second derivatives, quadratic functions, Euler's method, and the derivative and applications of various models (polynomial, exponential, sine), along with basic integrals using substitution. A total of 543 students participated in the study: 284 students completed the test in the context of calculus, and 259 students completed it in the context of kinematics. Students were randomly assigned to take one version of the test or the other within each group.

Instruments

We used two isomorphic tests, each consisting of twelve items, in the context of calculus and kinematics, to evaluate the derivative and antiderivative concepts. The calculus test was based on a previous conceptual test, TUG-C (Dominguez et al., 2017), and the kinematics test was based on the new version of TUG-K (Zavala et al., 2017). In **Appendix A**, we present the two tests used. **Figure 1** shows item 1 for both tests.

Table 1 describes both tests used in this study. It describes the four dimensions of the tests, the items contained in each dimension, the concepts evaluated, and the specific relationship evaluated.

Dimension 1 and dimension 2 of both tests are related since both assess the understanding of the derivative as slope. Dimension 3 and dimension 4 are also related since both assess the understanding of the antiderivative as the area under the curve. Our approach to asking for the antiderivative is in accordance with the accumulated change concept (Jones, 2015; Thompson & Silvermann, 2008). The difference in these related dimensions is the relationship that is evaluated. **Table 2** shows a detailed description of the items.

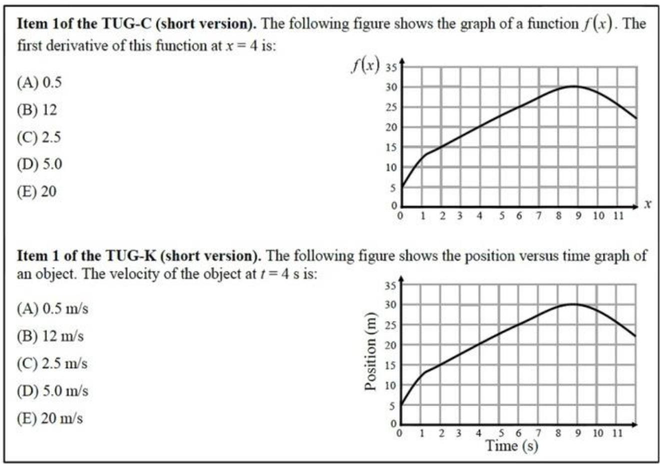

Figure 1. Item 1 of both tests used in this study (Source: Authors' own elaboration, see **Appendix A**)

Table 2 shows that the related dimensions items were assessed in the same way as the concept that is evaluated. The three items of the related dimension 1 and dimension 2 ask:

(1) to determine the positive value of a derivative,

(2) to determine the negative value of the derivative, and

(3) to identify the interval in which the derivative is the most negative.

The three items of the related dimension 3 and dimension 4 ask:

(1) to establish the procedure to determine the change of an antiderivative,

(2) to determine the value of the change of an antiderivative, and

(3) to identify the variable whose antiderivative has the greatest change in a specific interval.

Note that the first two items of each of the dimensions focus on obtaining a variable's value, and the third item focuses on finding a maximum of this variable.

Finally, it is important to establish why it was decided to shorten the versions of the tests used in this study. Our original version of the TUG-C has 16 items, while the tests used in this study have 12 items, so four

Table 1. Description of the isomorphic tests (kinematics/calculus version) used in this study by its dimension

Dimension	Description	Items	Concept	Relationship evaluated
1	Determine *f′(x)/ velocity* from the graph of *f(x)/position*	1, 4, & 10	The derivative as the slope	*f(x) → f′(x) / Position → velocity*
2	Determine *f′′(x)/acceleration* from the graph of *f′(x)/velocity*	5, 8, & 2		*f′(x)→ f′′(x) / Velocity → acceleration*
3	Determine the *change of f(x)/the change of position* from the graph of *f′(x)/velocity*	3, 11, & 12	The antiderivative as the area under the curve	*f′(x) → Δf(x) / Velocity → Δ position*
4	Determine the *change of f′(x)/the change of velocity* from the graph of *f′′(x)/acceleration*	9, 6, & 7		*f′′(x)→ Δf′(x) / Acceleration → Δ velocity*

Table 2. Description of the items of the isomorphic tests (kinematics/calculus version) by its dimension

Dimension	Description of the items
1	1. Determine the positive value of *f′(x)/the velocity* from the graph of *f(x)/position*
	4. Determine the negative value of *f′(x)/ the velocity* from the graph of *f(x)/position*
	10. Identify the interval in which *f′(x)/the velocity* is the most negative in the graph of *f(x)/position*
2	5. Determine the positive value of *f′′(x)/ the acceleration* from the graph of *f′(x)/velocity*
	8. Determine the negative value of *f′′(x)/ the acceleration* from the graph of *f′(x)/velocity*
	2. Identify the interval in which *f′′(x)/ the acceleration* is the most negative in the graph of *f′(x)/velocity*
3	3. Establish the procedure to determine the *change of f(x)/change of position* from the graph of *f′(x)/velocity*
	11. Determine the value of the *change of f(x)/change of position* from the graph of *f′(x)/velocity*
	12. Identify the *f(x) with the greatest change/ the object that has the greatest change in position* from several graphs of *f′(x)/velocity*
4	9. Establish the procedure to determine the *change of f′(x)/change of velocity* from the graph of *f′′(x)/acceleration*
	6. Determine the value of the *change of f′(x)/change of velocity* from the graph of *f′′(x)/acceleration*
	7. Identify the *f′(x) with the greatest change/ the object that has the greatest change in velocity* from several graphs of *f′′(x)/acceleration*

items were not included. It was decided not to include the four items where a graph is presented and asked to identify the corresponding graph (either its derivative or antiderivative) because being rigorous, these items can be answered by students using either the concept of derivative or the concept of the antiderivative.

RESULTS

Table 3 presents the results for the twelve items of the test. We present the percentages of students' answers for each option for each item in the calculus and kinematics tests. In **Table 3**, the correct option is bold, and there is a comparison between the results of students taking either the calculus or the kinematics test. We will discuss later whether or not there are significant differences between the answers.

Reliability and Discriminatory Power of the Tests

We perform the five statistical tests suggested by Ding et al. (2006) to evaluate the reliability and discriminatory power of the tests. Three measures

Table 3. Results of all the items

Item	Item description	Context	Percentage (%)				
			A	B	C	D	E
1	Determine the positive value of *f′(x)/the velocity* from the graph of *f(x)/position*	Calculus	12	6	**38**	26	18
			↕			↕	↕
		Kinematics	4	3	**38**	49	5
2	Identify the interval in which *f′′(x)/ the acceleration* is the most negative in the graph of *f′(x)/velocity*	Calculus	6	20	9	6	**58**
			][				↕
		Kinematics	3	14	7	2	**73**
3	Establish the procedure to determine the *change of f(x)/change of position* from the graph of *f′(x)/velocity*	Calculus	5	**60**	29	4	2
			↕	↕	↕	↕	
		Kinematics	1	**71**	16	11	1
4	Determine the negative value of *f′(x)/ the velocity* from the graph of *f(x)/position*	Calculus	**49**	8	8	22	13
				][	][	↕	↕
		Kinematics	**57**	4	3	34	2
5	Determine the positive value of *f′′(x)/ the acceleration* from the graph of *f′(x)/velocity*	Calculus	**45**	5	24	15	11
					↕	↕	↕
		Kinematics	**51**	3	39	5	2
6	Determine the value of the *change of f′(x)/change of velocity* from the graph of *f′′(x)/acceleration*	Calculus	14	15	11	**50**	10
		Kinematics	11	17	7	**53**	13
7	Identify the *f′(x) with the greatest change/ the object that has the greatest change in velocity* from several graphs of *f′′(x)/acceleration*	Calculus	10	**29**	4	29	27
			][				
		Kinematics	17	**30**	2	27	25
8	Determine the negative value of *f′′(x)/ the acceleration* from the graph of *f′(x)/velocity*	Calculus	15	**52**	9	14	10
			↕				↕
		Kinematics	24	**57**	7	10	2
9	Establish the procedure to determine the *change of f′(x)/change of velocity* from the graph of *f′′(x)/acceleration*	Calculus	**66**	20	6	5	3
					↕		
		Kinematics	**70**	18	2	4	6

Table 3 (Continued). Results of all the items

Item	Item description	Context	Percentage (%)				
			A	B	C	D	E
10	Identify the interval in which *f′(x)/the velocity* is the most negative in the graph of *f(x)/position*	Calculus	11	**61**	10	14	4
				↕	↕		
		Kinematics	10	**73**	3	12	2
11	Determine the value of the *change of f(x)/change of position* from the graph of *f′(x)/velocity*	Calculus	11	**50**	11	20	7
			↕	][	↕	↕	
		Kinematics	20	**60**	4	12	3
12	Identify the *f(x) with the greatest change/ the object that has the greatest change in position* from several graphs of *f′(x)/velocity*	Calculus	**24**	28	14	29	2
			][	↕			
		Kinematics	**34**	18	19	28	1

Note. The correct answer is in **boldface**; bold arrows (↕) indicate the options that have significant differences with $p < 0.01$ on Chi-square test; & double-bracket symbols (][) indicate the options that have significant differences with $0.05 < p < 0.01$

Table 4. Item difficulty index (P), item discriminatory index (D), and point-biserial coefficient (r_{pbs}) for each item of the calculus (TUG-C) and kinematics (TUG-K) isomorphic tests

TUG-C (short version)												
	1	2	3	4	5	6	7	8	9	10	11	12
P	0.38	0.58	0.60	0.49	0.45	0.50	0.29	0.52	0.66	0.61	0.50	0.24
D	0.54	0.72	0.55	0.86	0.85	0.87	0.62	0.65	0.75	0.59	0.77	0.68
r_{pbs}	0.50	0.55	0.42	0.64	0.66	0.64	0.55	0.53	0.53	0.50	0.59	0.63
TUG-K (short version)												
	1	2	3	4	5	6	7	8	9	10	11	12
P	0.38	0.73	0.71	0.57	0.51	0.53	0.30	0.57	0.70	0.73	0.60	0.34
D	0.71	0.56	0.65	0.83	0.88	0.71	0.66	0.74	0.71	0.54	0.83	0.69
r_{pbs}	0.59	0.44	0.58	0.67	0.72	0.58	0.54	0.59	0.62	0.42	0.66	0.57

Table 5. Summary of the results of the test statistics for the calculus (TUG-C) and kinematics (TUG-K) tests

Test statistic	Desired values	TUG-C (short version)	TUG-K (short version)
Difficulty index	[0.3, 0.9]	Average: 0.49	Average: 0.56
Discriminatory index	≥ 0.3	Average: 0.70	Average: 0.71
Point-biserial coefficient	≥ 0.2	Average: 0.56	Average: 0.58
Kuder-Richardson reliability index	≥ 0.7 for group measures	0.80	0.83
Ferguson's delta test	>0.9	0.99	0.99

examine individual test items: difficulty index, discriminatory index, and point biserial.

Table 4 presents these values for each item on the two tests. The other two measures examine the test as a whole: Kuder-Richardson's reliability test and Ferguson's delta test. We discuss the results of these statistical tests below.

The item difficulty index (P) measures the difficulty of a single item, and the criterion is that this index should be between 0.3 and 0.9 (Ding et al., 2006). As shown in **Table 4**, only two items on the TUG-C, item 7 (0.29) and item 12 (0.24), have indexes slightly lower than the desired, and no item on the TUG-K has an index lower than the desired. Researchers also suggest the calculation of the average value. The average difficulty values for the TUG-C and TUG-K are 0.49 and 0.56, respectively (**Table 5**), which fall into the suggested range [0.3-0.9].

The item discriminatory index (D) measures each item's discriminatory power, and the criterion is that this index should be above 0.3 (Ding et al., 2006). All the items of both tests fulfill this criterion (using the 25%-25% method). Investigators also recommend calculating the average value. For the TUG-C and the TUG-K, the average discriminatory values are 0.70 and 0.71, respectively, which meet the criterion (above 0.3).

The point-biserial coefficient (r_{pbs}) measures the consistency of a single item in relation to the whole test, and the criterion is that it should be above 0.2 (Ding et al., 2006). All the items of both tests fulfill this criterion. Researchers also recommend the calculation of the average coefficient. For the TUG-C and the TUG-K, the average coefficient values are 0.56 and 0.58, respectively, which fulfill the criterion (above 0.2).

Next, we focus on the two measures that examine the tests as a whole. Kuder-Richardson´s reliability test measures the self-consistency of the test, and the criterion is that this value should be above 0.7 for group measures (Ding et al., 2006). The indexes for the TUG-C and TUG-K are 0.80 and 0.83, respectively (**Table 5**), which meet this criterion. Furthermore, Ferguson's delta test measures the discriminatory power of the test, and

Table 6. Correct answer percentages of the items of the related dimension 1 and dimension 2, understanding of the concept of the derivative as slope

Description of items that evaluate the concept of derivative	Dimension 1			Dimension 2		
	Item	TUGC	TUGK	Item	TUGC	TUGK
Determine the positive value of a derivative	1	38%	38%	5	45%	51%
Determine the negative value of a derivative	4	49%	57%	8	52%	57%
Identify the interval in which the derivative is the most negative	10	61%	73%	2	58%	73%
Average		49%	56%		52%	60%

Table 7. Correct answer percentages of the items of the related dimension 3 and dimension 4, understanding of the concept of antiderivative as the area under the curve

Description of items that evaluate the concept of antiderivative	Dimension 3			Dimension 4		
	Item	TUGC	TUGK	Item	TUGC	TUGK
Establish the procedure to determine the change of an antiderivative	3	60%	71%	9	66%	70%
Determine the change of an antiderivative	11	50%	60%	6	50%	53%
Identify the variable whose antiderivative has the greatest change in a specific interval	12	24%	34%	7	29%	30%
Average		45%	55%		48%	51%

the criterion is that this value should be above 0.9 for good discrimination. Ferguson's delta test for both tests is 0.99, which satisfies this requirement. Finally, we show a summary of the five statistical tests in **Table 5**. Based on these analyses, we can conclude that both tests are reliable instruments with satisfactory discriminatory power.

Students' Overall Performance on Both Tests

The average score of the calculus test (TUG-C short version) is 5.83 out of 12 possible (each test item is worth 1 point). The distribution of these scores is not normal (D (284) = 0.1, $p < 0.01$). For this type of distribution, using the quartile as the measure of spread is preferable. The distribution median is 6, the bottom quartile (Q1) is 3, and the top quartile (Q3) is 8, so the interquartile range is 5. On the other hand, the average score of the test of kinematics (TUG-K short version) is 6.63. The distribution of these scores is also not normal (D (259) = 0.1, $p < 0.01$). The median of this distribution is 6, the bottom quartile (Q1) is 4, and the top quartile (Q3) is 10, so the interquartile range is 6.

Since neither distributions of scores were normal and their variances met the assumption of homogeneity of variance ($p < 0.05$; in a non-parametric Levene's test, Nordstokke and Zumbo (2010) and Nordstokke et al. (2011)), we decided to perform the comparison of both distributions using the nonparametric Mann-Whitney test (Field, 2013). This test indicates that the scores obtained by students in the kinematics test were significantly higher than those obtained by students in the test of calculus, U = 42109.0, p = 0.003, r = 0.13. Comparing the average scores in both tests (5.83 in calculus and 6.63 in kinematics), we can establish that the difference in average is about one question of the twelve questions.

Students' Performance in the Dimensions of Both Tests

Table 6 and **Table 7** show the correct answer percentages of the items grouped in each of the four dimensions and the averages of these percentages. Next, we analyze the students' performances.

As noted in **Table 6** and **Table 7**, the average percentages of the four dimensions of the two tests are very close, ranging from 45% to 60%. Also, we note that the score is greater in all the averages in the context of kinematics. In dimension 1, the difference is 7%, in dimension 2, 8%, in dimension 3, 10%, and finally, in dimension 4, 3%.

Dimension 1 and dimension 2 and dimension 3 and dimension 4 are directly related as the first evaluates the concept of the derivative as slope, while the latter evaluates the concept of the antiderivative as the area under the curve. As mentioned above, the difference between these dimensions is only in the relationship that is evaluated. Interestingly, the averages obtained in these related dimensions are very similar for each test. In the calculus test, the average percentages of dimension 1 and dimension 2 are 49% and 52%, respectively, while the percentages in dimension 3 and dimension 4 are 45% and 48%. Moreover, in the kinematic test, dimension 1 and dimension 2 average percentages are 56% and 60%, while those in dimension 3 and dimension 4 are 55% and 51%. As seen in all cases, the averages have a difference of less than 5%. These results show that the overall understanding of students on the concepts of derivative and antiderivative are similar in each of the contexts in the two relationships that are evaluated.

Overview of the Effect of Context on Performance in Each of the Test Item

We use the Chi-square test to compare students' performances and detect significant differences.

Table 8. Classification of items according to the differences found in the selection of the correct and incorrect answers

Division of items by the selection of the correct answer	Group of items	Items	Correct answer (calculus)	Correct answer (kinematics)	General trends detected in the selection of incorrect answers
Items with significant differences in the selection of the correct answer	First group: Items of dimension 1 and dimension 2 that evaluate the identification of the interval in which the derivative is the most negative	10	61%	73%	Trends detected
		2	58%	73%	
	Second group: All the items of dimension 3	3	60%	71%	Trends detected
		11	50%	60%	
		12	24%	34%	
No significant differences in the selection of the correct answer	Third group: Items of dimension 1 and dimension 2 that evaluate the determination of the value of a derivative	1	38%	38%	Trends detected
		4	49%	57%	
		5	45%	51%	
		8	52%	57%	
	Fourth group: All the items of dimension 4	9	66%	70%	No trends detected
		6	50%	53%	
		7	29%	30%	

According to Sheskin (2007), detecting significant differences requires two steps using the Chi-square test. The first one uses the Chi-square test ($p < 0.05$) to determine whether there was a significant difference in the distribution of answers between the two problems. If this difference does exist, it is followed by the second step. In this latter step, the Chi-square test is used to determine which specific option is significantly different. Sheskin (2007) mentions that there is no consensus on the p-value for this second step. Some researchers recommend using the Bonferroni correction, dividing the original p-value (0.05) by the total number of comparisons (five, one for each multiple choice) for a conservative p-value of 0.01, thus avoiding inflating the error type I. On the other hand, other researchers consider that this correction is extremely severe since it substantially reduces the power associated with each comparison and that, in the final analysis, one must decide what p-value (per comparison) results on a fair balance in terms of the likelihood of committing the type I error and the power associated with a comparison.

Because of this lack of consensus for the second step, in this study, we decided to identify the options separately with a significant difference with a p-value less than 0.01 and options with a significant difference with a p-value less than 0.05 but greater than 0.01. It is important to mention two notes about this. First, most of the differences detected in this study are less than 0.01 (22 out of the 28 significant differences detected in the 12 items are below this value). Second, in this study, we focus on the significant differences repeated in the related items and identify them as trends. **Table 8** shows a classification of items according to these trends.

As seen in **Table 8**, we clustered the items by significant differences detected in selecting the correct answer. In **Table 8**, we identify two groups of related items (first and second group), and we find that students in the kinematics context choose the correct answer in a greater proportion than those students who choose the correct answer in the calculus context. On the other hand, in **Table 8**, we identify two other groups of related items (third and fourth group), where we find no differences in selecting the correct answer. We did not find any item in which students in the calculus context chose the correct answer in a greater proportion than those students who chose the correct answer in the kinematics context. Finally, in **Table 8**, we also establish whether we find general trends in selecting incorrect answers in the four groups of items. Next, we analyze the items in **Table 8**. We analyze:

(1) the differences in the selection of the correct answers,

(2) the most frequent errors, and

(3) the differences in the selection of the incorrect answers.

Items With Significant Differences in the Selection of the Correct Answer

We analyzed the two item groups and found significant differences in selecting the correct answer.

First group: Items of dimension 1 and dimension 2 that evaluate the identification of the interval in which the derivative is the most negative

Table 3 and **Table 7** show the results of the two items of the first group of dimension 1 and dimension 2 (item 10 and item 2) that evaluate the identification of the interval in which the derivative is the most negative. Item 10 asks to identify the interval in which the first derivative or the velocity is the most negative, and item 2 asks to identify the interval in which the second derivative or the acceleration is the most negative. (Note that in both items the slopes are constant in all the intervals on the graphs.) In both items, the percentage of correct answers is significantly higher in kinematics than in calculus (item 10: 73% vs. 61%; item 2: 73% vs. 58%).

Table 3 shows that the most common error in item 10 for both contexts is selecting option D (14% in calculus and 12% in kinematics), while in item 2, the most frequent error is selecting option B (20% and 14%, respectively). In both items, these errors involve choosing an interval where the derivative is negative but not the most negative. This pattern is consistent across both contexts, with no significant difference between the groups of students who made this error. The primary issue in students' understanding appears to be identifying the steepest slope rather than recognizing the sign of the slope. Additionally, students' choices may reflect a naïve misconception–often called slope-height confusion–since the selected interval is the only one on the graph with negative values on the y-axis.

We also observe that in both items, the context of calculus triggers the incorrect answer in which students identify the interval requested with the interval in which the derivative is the most positive instead of the most negative (option C in item 10: 10% vs. 3%; option A in item 2: 6% vs. 3%). It is important to mention that these differences are from percentages equal to or lower than 10%. This fact reduces, in some way, the importance of the result. However, since that interval has the greatest absolute value rate of change, it might not be clear to these students that the slope has a sign in calculus. With significantly less proportion that happens to students who took the test in kinematics.

Second group: Items of dimension 3

Table 3 and **Table 7** show the results of the three items of the second group (items 3, 11, and 12). These are all the items of dimension 3 that ask to determine the change of *f(x)* from the graph of *f′(x)* (in calculus) or the change of position from the graph of velocity (in kinematics). The percentage of correct answers in the three items is significantly higher in kinematics than in calculus (item 3: 71% vs. 60%; item 11: 60% vs. 50%; item 12: 34% vs. 24%).

From **Table 3**, we can also analyze the most frequent errors in the three items. Item 3 asks students to establish the procedure to determine a change of *f(x)* in an interval from the graph of *f′(x)* or a change of position in an interval from the velocity graph (note that the slope of the curve is constant in intervals). The most frequent error in item 3 for both contexts is option C. In this error, students establish that calculating the requested change is the procedure to calculate the slope of the curve instead of the area under the curve. The most frequent incorrect answer in the kinematics test is option D. In the case of calculus, 4% of students chose that answer, too. However, the 11% in the case of kinematics is significantly higher. To understand this difference, analyzing the resources students use is necessary. The change of position is asked in the kinematics problem, and students who choose this common error in kinematics might incorrectly use the equation $d = vt$ as a resource (Beichner, 1994). Students might think that if they have the velocity (5 m/s) and time (2 s), they can calculate distance by multiplying those two.

Item 11, in the context of calculus, asks to calculate the change of *f(x)* from the graph of *f′(x)* in the interval from $x = 0$ to $x = 4$. In the context of kinematics, the problem asks to find the change of position from the velocity graph in the interval from $t = 0$ to $t = 4$ s. (Note that the slope of the curve is also constant in the interval). The most frequent error in this item is different in the two contexts. In the context of calculus, the most frequent error is to choose the value of the slope of the curve instead of the value of the area under the curve (option D). On the other hand, in the context of kinematics, the most frequent error (option A) is to choose the value ('20') that is obtained by multiplying the horizontal change in the interval (which is 4) by the vertical change in the interval (which is 5). Like what happened with item 3, students might use the equation $d = vt$ as a resource. The fact that this error is not the most common in the same context in item 3 appears to be because item 11 explicitly calls for a value (not a procedure). This seems to make students in the kinematics context analyze by using formulas as resources. In the case of calculus, there is no interpretation like that in kinematics in which students relate. In this case, the most common incorrect answer for item 11 is the same as the most common incorrect answer for item 3, which is related to interval-point confusion. Another important factor to consider when comparing item 3 and item 11 is that the options listed in item 3 appear two terms that can greatly influence students' responses. These terms are "area" (in the correct answer) and "slope" in one of the most common wrong answers. Therefore, in the TUG-C, the term "slope" was very attractive to students; instead, the distance formula $d = vt$ was more attractive in TUG-K.

Item 12 asks to identify the *f(x)* with the greatest change from several graphs of *f′(x)* or the object that has the greatest change in position from several graphs of velocity. The most frequent error in item 12 for both contexts is option D. In this error students do not choose the graph with the curve with the greatest area under it (option A), but a graph with a curve whose slopes in the interval are always positive and increasing. In this item students seem also to be thinking regarding slope in both contexts. That is, this item asks for a function with the greatest change in the interval and students choose the curve that has the greatest change in positive slopes. Note that option B is an option with the greatest change in slope (goes from positive, to zero and negative values) and it is the very attractive answer in the context of calculus, but less in kinematics; that is why it is discussed below as a trend.

As seen in **Table 3**, in the items of this group we also detected significant differences in the selection of incorrect options. In item 3 and item 11 that ask explicitly to determine the change in a function, we found three tendencies. The first tendency is that the context of calculus triggers the selection of the value of the slope of the curve instead of the value of the area under the curve (option C in item 3: 29% vs. 16%; option D in item 11: 20% vs. 12%). The second tendency is that the context of calculus triggers the selection of the vertical value of the curve at the right point of the interval (option A in item 3: 5% vs. 1%; option C in item 11: 11% vs. 4%). The third tendency is that the context of kinematics triggers the selection of the value that is obtained by multiplying the horizontal change in the interval by the vertical change in the interval (option D in item 3: 11% vs. 4%; option A in item 11: 20% vs. 11%). As mentioned, these latter students use the equation $d = vt$ as a resource.

In item 12 we found only one tendency in selecting an incorrect option. The context of calculus triggers the selection of a symmetrical concave down graph with a curve that increases in the first half of the interval and decreases in the other half (option B: 28% vs. 18%). This trend can be understood from the two first tendencies detected in item 3 and item 11 (slope and y-value). The first is that the context of calculus triggers the selection of the slope value. In item 12, students also seem to be thinking about slope. This item asks for a variable with the greatest change in the interval and students choose the curve with the greatest change in slope as it begins in a high positive value and ends at the same value but negative. The second trend is that the context of calculus triggers the reading of the y-value. Students seem to think this curve has the greatest change in the y-value since it goes from zero to a high value and then decreases to zero again (as we have seen in student interviews).

Items With No Significant Differences in the Selection of the Correct Answer

Next, we analyze the two groups of items that meet this feature: no significant differences in selecting the correct answer.

Third group: Items of dimension 1 and dimension 2 that evaluate the calculation of the value of a derivative

Table 3 and **Table 7** show the results of the four items of the third group (items 1, 4, 5, and 8) of dimension 1 and dimension 2 that evaluate the determination of the value of a derivative at a point of a curve. Item 1 and item 4 ask to determine the value of the first derivative or the velocity (positive and negative value, respectively), and item 5 and item 8 ask to determine the value of the second derivative or the acceleration (positive and negative value, respectively). In the items of this group, we detected no significant differences in the selection of the correct answer.

From **Table 3**, we can also analyze the most frequent errors in the four items of this group. It is very interesting to note that in the four items for both contexts, the most common error is the same. As mentioned before, these items evaluate the determination of the value of a derivative at a point of a curve. The most frequent error is to obtain this value by dividing the vertical value of the curve at the point by the horizontal value of the point in situations where this is not applicable (option D in item 1 and item 4, option C in item 5, and option A in item 8). It is important to note that in the items that ask for a negative derivative (item 4 and item 8), students add a negative sign to the value obtained in this division.

As seen in **Table 3**, we detected significant differences in the selection of incorrect options for the items of this group. Analyzing these differences globally shows that only two significant differences are repeated in all group items. These differences can be identified as general tendencies and due to this fact, have great instructional value. We decided to focus on these tendencies in this study. The first tendency is that the context of kinematics triggers the selection of the option in which students divide the vertical value of the curve at the point by the horizontal value of the point in situations where this is not applicable (item 1 and item 4 option D; item 5 option C; item 8 option A). As mentioned, this is the most frequent error of all the items. To understand this, it is necessary to analyze the resources students use. The kinematics problem is asked to determine the velocity or the acceleration, and students who choose this option in kinematics seem to use the equations $v = d/t$ or $a = v/t$ as resources.

The second trend is that the context of calculus triggers the selection of the option in which students choose the vertical value of the curve at the point instead of the slope value at the point (option E items 1, 4, and 8; option D item 5). We can see that it is more common in calculus for students to identify the value of the first derivative at a point with the value of the function at that point than in kinematics. In this latter case, students identify the velocity value at a specific time with the position's value at that specific time (item 1 and item 4). We can also note that it is more common for calculus students to identify the value of the second derivative at a point with the value of the first derivative at that point than in kinematics. In this case, students identify the value of the acceleration at a specific time with the value of the velocity at that specific time (item 5 and item 8).

Fourth group: Items of dimension 4

Table 3 shows the results of the three items of the fourth group (items 9, 6, and 7). These items are from the fourth dimension and ask to determine the change of $f'(x)$ from the graph of $f''(x)$ (in calculus) or the change of velocity from the graph of acceleration (in kinematics).

In these items, we detected no significant differences in selecting the correct answer.

From **Table 3**, we can analyze the most frequent errors in the three items of this group. It is very interesting to note that in the three items for both contexts, the most common error is the same. Students identified the requested value (area under the curve) with the slope value. In item 9, students establish the procedure to obtain a slope (option B). In item 6, students obtain the value of a slope (option B). Finally, in item 7, students think about slopes and choose the curve with the greatest change in positive slopes (option D) instead of the curve with the greatest area under it.

As shown in **Table 3**, in this group, we did not observe general trends in selecting incorrect answers. The lack of tendencies contrasts with what we detected in the other three groups, where we observed the general trends described above. In this group, we only observed two significant differences isolated in two items. In item 9, the context of calculus triggers the selection of the vertical value of the curve at the right point of the interval instead of the area under the curve (option C). Additionally, in item 7, the context of kinematics triggers the selection of a curve with a constant positive slope instead of a curve with a constant value and maximum area under it (option A). These trends are isolated, and it is important to note that the first has a difference of very low percentages (6% vs. 2%), and the second trend has a p-value that is not less than 0.01. As mentioned, it was decided to focus solely on the general trends due to its greater instructional value.

DISCUSSION

We discuss the main findings of this study relating them with previous studies (Carli et al., 2020; Ivanjek et al., 2016; Planinic et al., 2013) described in Section 2. Additionally, we establish possible future studies on each topic.

Effect of Context on Students' Scores

We concluded that students scored significantly higher on the kinematics test than the calculus test (objective 1). When comparing the average scores between the two tests–5.83 for calculus and 6.63 for kinematics–the difference amounts to approximately one question out of twelve. This general finding can be further understood by analyzing student performance on individual test items (objective 3). As discussed earlier, we found that for two groups of items, a greater proportion of students selected the correct answer in the kinematics context. This likely explains the overall higher performance of students in the kinematics test.

Our study found that students performed better in the kinematics context, which contrasts with previous research (Carli et al., 2020; Ivanjek et al., 2016; Planinic et al., 2013), where higher scores were observed in the mathematics context. As discussed before, a key difference between those studies and ours lies in how the questions were framed in the context of mathematics. This variation in question phrasing likely explains the discrepancy between our findings and previous research findings. Our aim was to provide a more comprehensive and isomorphic comparison between the two contexts, as outlined earlier. Future research should further explore how question phrasing affects students' graphical understanding of these concepts.

Effect of Context on Students' Performance on Each Item

In this section, we present a synthesis of the effect of the context on students' performance on each item of the tests (related to objective 2 and objective 3), focusing on related dimensions. The items from the two tests were concentrated into four groups (**Table 7**).

The first and third groups of items correspond to dimension 1 and dimension 2, which assess students' understanding of the derivative as slope. In the third group of items, which focus on determining whether the value of the derivative at a point is positive or negative, we did not observe any significant context effect on students' ability to select the correct answer. However, in the first group of items, which requires identifying the interval where the derivative is most negative, students performed significantly worse in the calculus context compared to the kinematics context. Overall, these findings suggest that students have a similar understanding of how to determine positive and negative derivative values in both contexts. However, they struggle to identify the interval of the greatest negative slope in the calculus context.

The second and fourth groups consist of items from dimension 3 and dimension 4, which assess students' understanding of the antiderivative as the area under a curve. In the second group, items from dimension 3 evaluate the calculation of the change in $f(x)$ from the graph of $f'(x)$ (in calculus) or the change in position from the graph of velocity (in kinematics). Here, we observed significantly higher performance in the kinematics context. In contrast, in the fourth group, which includes items from dimension 4 that assess the determination of the change in $f'(x)$ from the graph of $f''(x)$ (in calculus) or the change in velocity from the graph of acceleration (in kinematics), there was no significant context effect on students' selection of the correct answer.

Next, we compare our findings with previous studies' findings (Carli et al., 2020; Ivanjek et al., 2016; Planinic et al., 2013). First, we focus on the results related to students' understanding of the concept of slope. Planinic et al. (2013) and Ivanjek et al. (2016) concluded that students' understanding of slope is similar across mathematics and kinematics contexts. This finding aligns with one of our results regarding students' understanding of the derivative as a slope. Our study observed that students determined positive and negative derivative values similarly in both contexts. However, our study offers two significant contributions to this discussion. First, our comparison was aimed to be more comprehensive, as we evaluated two different relationships in both contexts, as outlined in Section 2. Second, we found that students had greater difficulty identifying the interval where the derivative is most negative in the calculus context, a challenge not highlighted in previous research.

We can also compare our results on the concept of slope with those of Carli et al. (2020). Their study includes one item (question 3) closely related to our derivative items. In that question, students are presented with a graph and asked to determine the value of the "first derivative" in the calculus context and the value of "velocity" in the kinematics context. This is similar to our item 1, which asks students to determine the positive value of a derivative. Unlike our findings, Carli et al. (2020) reported that students performed better on the calculus item. As mentioned before, their participants were university students who had completed their first calculus course but had not yet taken their first physics course. In contrast, our participants had completed both their first calculus and physics courses. We believe this difference in participants' academic backgrounds may account for the discrepancy in results. Additionally, our study aimed to offer a more comprehensive comparison, as outlined before. First, we included two additional items (item 4 and item 10) to evaluate understanding of the derivative. Second, we assessed this concept by exploring both possible relationships between the derivative and the context.

We now turn to comparing the results regarding students' understanding of the area under the curve. Planinic et al. (2013) and Ivanjek et al. (2016) found that students performed better on problems asking for the area under the curve in the context of mathematics than on similar problems in the context of kinematics. At first glance, this seems inconsistent with our findings. However, a closer analysis reveals the underlying differences. In our study, students performed significantly better on items asking them to determine the change in position from a velocity graph (kinematics) compared to items asking for the change in *f(x)* from the graph of *f'(x)* (calculus). Additionally, we found no significant difference in students' performance on items asking for the change in velocity or the change in *f'(x)*. A detailed comparison of the items used by Planinic et al. (2013) and Ivanjek et al. (2016) shows that their questions explicitly asked students to find the "area," which may have been easier for university students. This likely explains the better performance in mathematics observed in their studies.

We can also compare our findings related to the area under the curve with those of Carli et al. (2020). The researchers included an item (question 9) that aligns with our antiderivative items in their study. Their question asked students to determine the value of the "definite integral" in the calculus context and the value of "displacement" in the kinematics context. This is similar to our item 11, which asks students to determine the change of an antiderivative. Unlike our study, Carli et al. (2020) reported that students performed similarly across both contexts. We believe this difference may be attributed to the phrasing of the questions ("definite integral" in their study versus "change of *f(x)*" in ours), as well as the different participant groups. Additionally, we included two additional items (item 3 and item 12) to assess understanding of this concept and evaluated it by considering both possible relationships.

In future studies, it would be valuable to explore how the phrasing of questions affects students' understanding of these concepts across the two possible relationships. We recommend conducting these studies with participants who have completed both a calculus course and a mechanics course, which will cover these topics. Additionally, in our study, we observed that students performed significantly better on items asking them to determine the "change of position" (first relationship, kinematics) compared to those asking for the "change of *f(x)*" (calculus). However, no significant difference was found in performance on items assessing the "change of velocity" or "the change of *f'(x)*" (second relationship). Further research is needed to understand why responses related to these two relationships did not exhibit the same behavior patterns.

Most Frequent Errors and the Errors Triggered in Each Context

This section discusses the most frequent errors, and the errors triggered in each context (objective 3), focusing on related dimensions.

The first and third groups of items assess students' understanding of the derivative as slope (dimension 1 and dimension 2). The third group focuses on calculating the value of a derivative (positive and negative) at a specific point. The most common error observed in this group is related to interval-point confusion, where students incorrectly divide the vertical value of the curve at a point by the horizontal value, even in situations where this approach is not applicable. We identified two general tendencies in students' incorrect answer choices:

(1) In the kinematics context, students are more likely to select the most frequent error, possibly due to reliance on equations such as $v = d/t$ or $a = v/t$.

(2) In the calculus context, students tend to choose the vertical value of the curve at the point, reflecting the slope-height confusion.

When comparing our findings to previous studies, Carli et al. (2020) reported results consistent with both of these trends, while Ivanjek et al. (2016) observed only the first trend. It is important to note that the latter study asked students to find the "slope" rather than the "derivative."

The items in the first group assess students' ability to identify the interval where the derivative is most negative. The most common error occurs when students select an interval where the derivative is negative but not the most negative. Additionally, we observed a specific trend in the calculus context: students often choose the interval where the derivative is most positive, rather than the most negative, when asked to identify the correct interval. This particular trend was not found in previous studies, as similar types of questions were not included in their analyses.

The six items in the second and fourth groups evaluate students' understanding of the antiderivative as the area under a curve (dimension 3 and dimension 4). In most items from these groups, the most frequent error is confusing the requested value (area under the curve) with the slope of the graph. Furthermore, in the three items from the second group (dimension 3), we identified three distinct trends in the selection of incorrect answers.

The first two general tendencies observed in the three items are, as follows: in the calculus context, students often mistake the change in the function (represented by the area under the curve within an interval) for either the slope of the curve (trend 1) or the vertical value of the curve at the right endpoint of the interval (trend 2). These two tendencies may be interrelated. A common misconception is that the change in a function is directly linked to its rate of change, leading students to attempt to calculate the slope over the interval. Some successfully compute the slope, while others encounter an additional issue, known as slope-height confusion. When comparing our findings to previous studies, Carli et al. (2020) reported similar results to our trend 2 but did not identify trend 1. It's important to note that they asked about the "definite integral," whereas we asked about the "change of *f(x)*". Additionally, these tendencies were not reported by Ivanjek et al. (2016).

The third trend, found in two of the three items, is that students tend to select the value obtained in the kinematics context by multiplying the horizontal change in the interval by the vertical change. This can be explained by the fact that students often rely on the equation $d = vt$ as a resource. Ivanjek et al. (2016) identified this tendency as one of the significant errors students made in the physics context, while Carli et al. (2020) did not report it.

Finally, in future research, it would be valuable to investigate further how the phrasing of questions influences the most frequent errors and how they are triggered differently in each context.

CONCLUSIONS

In conclusion, this study highlights that the context in which graph-related concepts of derivatives and antiderivatives are taught significantly impacts students' understanding and performance. Students performed better in kinematics contexts, likely due to the tangible, real-world nature of physical examples compared to the more abstract mathematical problems. This finding emphasizes the role of context in education and suggests that incorporating more concrete, real-world examples into calculus instruction could enhance students' comprehension.

Despite stronger performance in kinematics, students still displayed persistent misconceptions, such as interval-point and slope-height confusion in calculus and the misapplication of formulas in kinematics. These results suggest the need for instructional strategies specifically targeting and correcting these errors. Educators should consider using contextually rich problems and providing clearer explanations to help address these common misunderstandings.

Future research should explore the impact of different questioning strategies and further investigate the underlying causes of context-specific performance differences. Expanding the study to include a more diverse range of student populations and instructional approaches could offer a broader understanding of effectively teaching these key concepts.

By focusing on these areas, educators can develop more effective teaching methods that improve students' conceptual understanding and enhance their ability to apply these concepts across various contexts. This, in turn, can help build a stronger and more adaptable mathematical foundation.

Instructional Recommendations

In this study, we found significantly lower student performance in the calculus context compared to the kinematics context in two specific areas:

(1) identifying the interval where the derivative is most negative and

(2) determining the change in an antiderivative.

These findings suggest that instructors should be mindful of students' challenges with this type of problem in the calculus context.

Additionally, we identified the most frequent errors for each item and pinpointed the incorrect options most commonly selected in each context. Instructors aiming to improve students' understanding of these concepts across both contexts should consider these insights when planning their instruction.

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APPENDIX A: TESTS USED IN THE STUDY

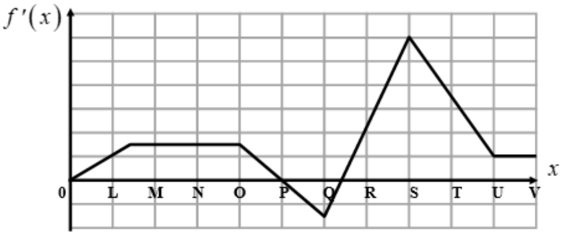
TUG-C Shorter Version

Notice that all the graphs in this questionnaire refer to a function *f* which depends on *x*, that is *f(x)*. Also, note that *f′(x)* corresponds to the first derivative of the function with respect to *x*, and *f′′(x)* is the second derivative of the function with respect to *x*. Moreover, *f(x)*, *f′(x)*, and *f′′(x)* are graphed with respect to *x*, this means *x* is the variable on the horizontal axis.

1. The following figure shows the graph of a function *f(x)*. The first derivative of this function at $x = 4$ is:

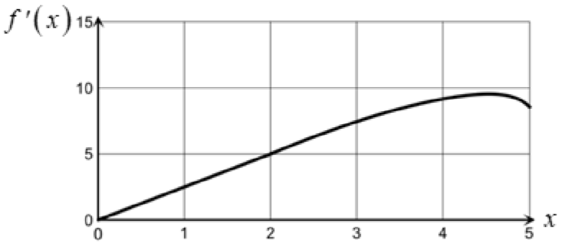
   a. 0.5
   b. 12
   c. 2.5
   d. 5.0
   e. 20

2. The following figure shows the graph of *f′(x)*, the first derivative of a function. Which of the following options corresponds to the case when the second derivative of the function is the most negative?



   a. Q to S
   b. O to Q
   c. At Q
   d. At S
   e. S to U

3. The following figure shows the graph of *f′(x)*, the first derivative of a function. If you wanted to know the change in the function *f(x)* in the interval from $x = 0$ to $x = 2$, from the graph you would:

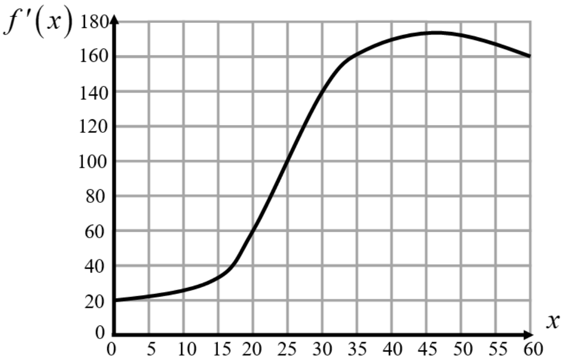


   a. Read 5 directly off the vertical axis.
   b. Find the area between the line and the horizontal axis by calculating 5*2/2.
   c. Find the slope of that line segment by dividing 5 by 2.
   d. Find the value multiplying 5 by 2.
   e. Find the value by dividing 2 by 5.

4. The following figure shows the graph of a function *f(x)*. The first derivative of this function at $x = 8$ is about:

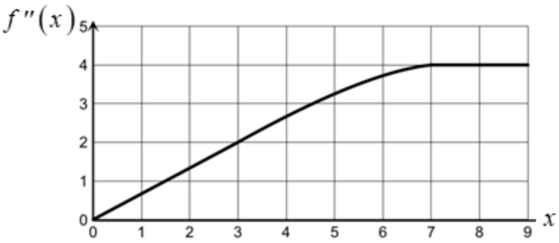
   a. -5.0
   b. -2.0
   c. -0.5
   d. -2.5
   e. 20

5. The following figure shows the graph of *f′(x)*, the first derivative of a function. At $x = 25$, the second derivative of the function is about:



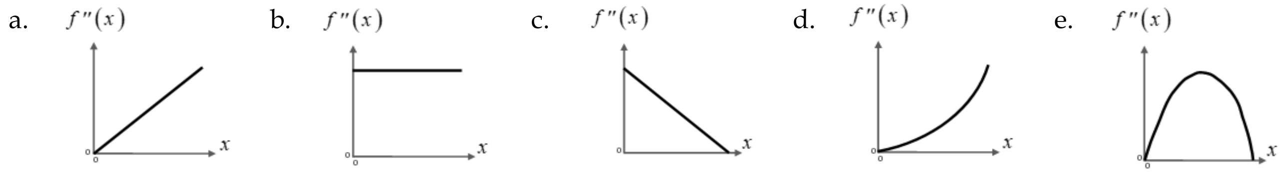
   a. 8.0
   b. 2.0
   c. 4.0
   d. 100
   e. 60

6. The following figure shows the graph of *f′′(x)*, the second derivative of a function. The change in the first derivative of the function in the interval from $x = 0$ to $x = 3$ is:

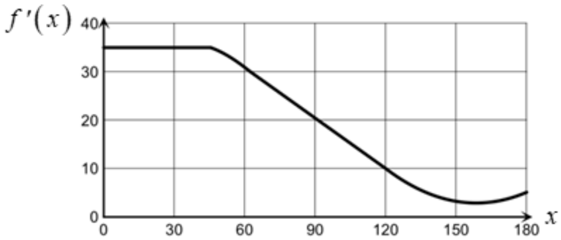


   a. 1.5
   b. 0.67
   c. 2.0
   d. 3.0
   e. 6.0

7. Graphs of the second derivative for five functions are shown below. All axes have the same scale. Which of the following graphs corresponds to the function with the greatest change in its first derivative in the interval?

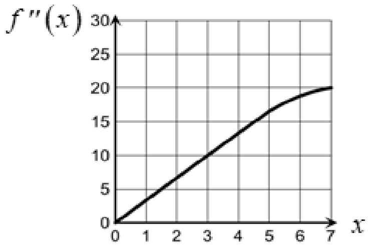


8. The following figure shows the graph of $f'(x)$, the first derivative of a function. At x = 90, the second derivative of the function is:



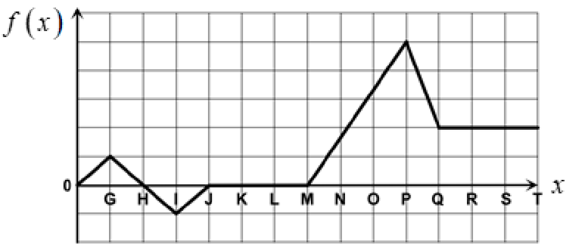
a. -0.22
b. -0.33
c. -1.0
d. -2.0
e. 20

9. The following figure shows the graph of $f''(x)$, the second derivative of a function. If you wanted to know the change in the first derivative of the function in the interval from x = 0 to x = 3, from the graph you would:



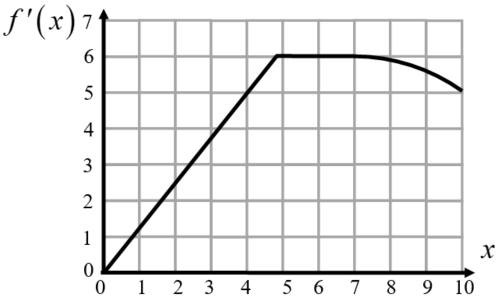
a. Find the area between the line and the horizontal axis by calculating 10*3/2.
b. Find the slope of that line segment by dividing 10 by 3.
c. Read 10 directly off the vertical axis.
d. Find the value by dividing 3 by 10.
e. Find the value by multiplying 10 by 3.

10. The following figure shows the graph of a function $f(x)$. Which of the following options corresponds to the case when the first derivative of the function is the most negative?



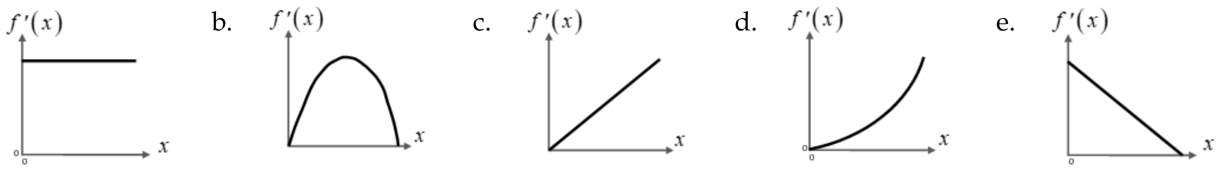
a. At I
b. P to Q
c. M to P
d. G to I
e. At P

11. The following figure shows the graph of *f′(x)*, the first derivative of a function. The change in the function *f(x)* in the interval from $x = 0$ to $x = 4$ is:



a. 20
b. 10
c. 5.0
d. 1.25
e. 0.8

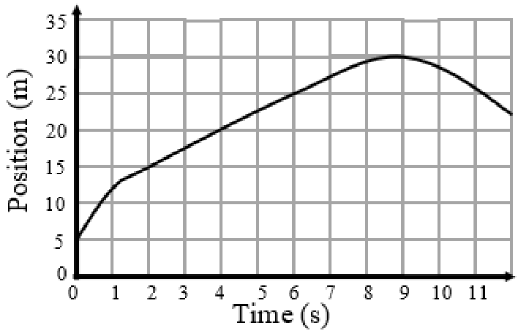
12. Graphs of the first derivative for five functions are shown below. All axes have the same scale. Which of the following graphs corresponds to the function *f(x)* with the greatest change in the interval?



TUG-K Shorter Version

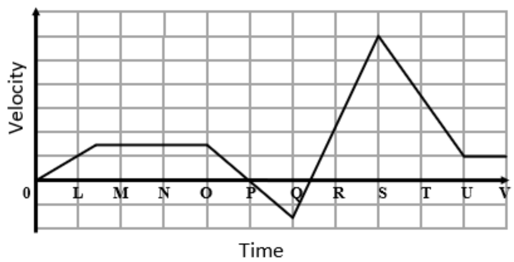
Notice that all the graphs in this questionnaire refer to the motion of an object along a straight line, that is, in one dimension. Moreover, the position, velocity and acceleration of the object are graphed with respect to time; this means time is the variable on the horizontal axis.

1. The following figure shows the position versus time graph of an object. The velocity of the object at $t = 4$ s is:



a. 0.5 m/s
b. 12 m/s
c. 2.5 m/s
d. 5.0 m/s
e. 20 m/s

2. The following figure shows the velocity versus time graph of an object. Which of the following options corresponds to the case when the acceleration of the object is the most negative?

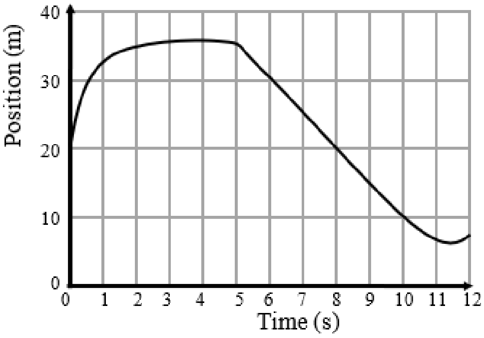


a. Q to S
b. O to Q
c. At Q
d. At S
e. S to U

3. The following figure shows the velocity versus time graph of an object. If you wanted to know the change in position of the object during the interval from $t = 0$ to $t = 2$ s, from the graph you would:

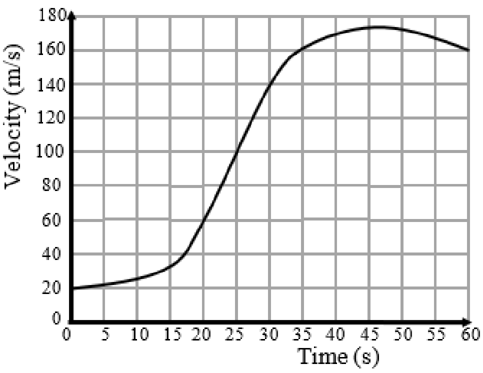
a. Read 5 directly off the vertical axis.
b. Find the area between the line and the horizontal axis by calculating 5*2/2.
c. Find the slope of that line segment by dividing 5 by 2.
d. Find the value multiplying 5 by 2.
e. Find the value by dividing 2 by 5.

4. The following figure shows the position versus time graph of an object. The velocity of the object at $t = 8$ s is about:



a. -5.0 m/s
b. -2.0 m/s
c. -0.5 m/s
d. -2.5 m/s
e. 20 m/s

5. The following figure shows the velocity versus time graph of an object. At $t = 25$ s, the acceleration of the object is about:



a. 8.0 m/s^2
b. 2.0 m/s^2

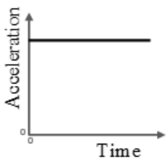
c. 4.0 m/s^2
d. 100 m/s^2
e. 60 m/s^2

6. The following figure shows the acceleration versus time graph of an object. The change in the velocity of the object during the interval from t = 0 to t = 3 s is:

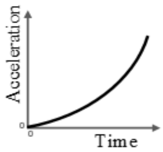
a. 1.5 m/s
b. 0.67 m/s
c. 2.0 m/s
d. 3.0 m/s
e. 6.0 m/s

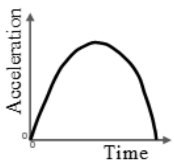
7. Acceleration versus time graphs for five objects are shown below. All axes have the same scale. Which of the following graphs corresponds to the object that has the greatest change in velocity during the interval?

a.

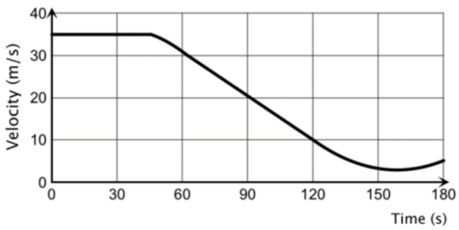

b.


c.


d.


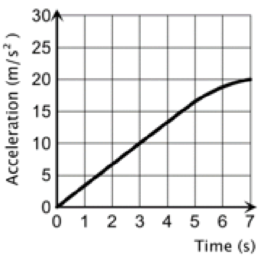
e.


8. The following figure shows the velocity versus time graph of an object. At t = 90 s, the acceleration of the object is:



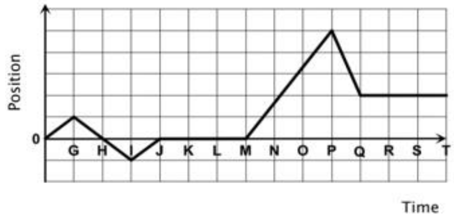
a. -0.22 m/s^2
b. -0.33 m/s^2
c. -1.0 m/s^2
d. -2.0 m/s^2
e. 20 m/s^2

9. The following figure shows the acceleration versus time graph of an object. If you wanted to know the change in velocity of the object during the interval from t = 0 to t = 3 s, from the graph you would:



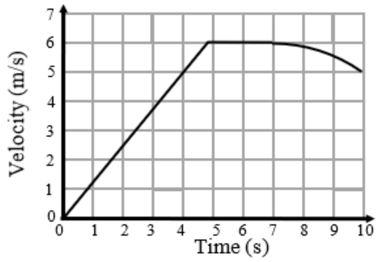
a. Find the area between the line and the horizontal axis by calculating 10*3/2.
b. Find the slope of that line segment by dividing 10 by 3.
c. Read 10 directly off the vertical axis.
d. Find the value by dividing 3 by 10.
e. Find the value by multiplying 10 by 3.

10. The following figure shows the position versus time graph of an object. Which of the following options corresponds to the case when the velocity of the object is the most negative?



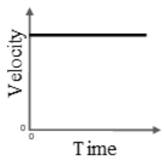
a. At I
b. P to Q
c. M to P
d. G to I
e. At P

11. The following figure shows the velocity versus time graph of an object. The change in the position of the object during the interval from t = 0 to t = 4 s is:



a. 20 m
b. 10 m
c. 5.0 m
d. 1.25 m
e. 0.8 m

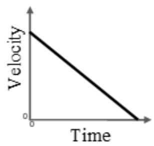
12. Velocity versus time graphs for five objects are shown below. All axes have the same scale. Which of the following graphs corresponds to the object that has the greatest change in position during the interval?

a.


b.


c.


d.


e.


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