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The contribution of critical thinking skills in rich mathematical problem completion: Insights from pre-service mathematics teachers

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Abstract

A narrow understanding of critical thinking challenges pre-service mathematics teachers to recognize situations that need critical thinking in problem-solving, remove errors in the formulation of mathematical problems due to unsupervised modification of data, and view content as a stimulus for critical thinking. This study aims to reveal how they perceive the extent to which rich mathematical problems require the contribution of critical thinking skills in their completion. We adopted grounded theory to locate the common trends of their perceptual following open, axial, and selective coding. Data saturation stemmed from the participation of 19 pre-service mathematics teachers through an in-depth interview session. Each pre-service mathematics teacher successfully recognized the application of at least two critical thinking skills in rich mathematical problem completion. The lack of understanding of critical thinking skills led them to focus on information as the catalyst for interpretation, while the question serves as the catalyst for evaluation and inference. They exhibited their involvement in critical thinking through questioning and discussion.

Keywords: critical thinking skills, perception, rich mathematical problems

INTRODUCTION

Pre-service mathematics teachers, as social actors, require professional competence development to navigate their profession and adjust to changes in the workplace through access to critical thinking and practical experiences during university education (Bezanilla et al., 2021). Critical thinking integration in teacher preparation programs equips pre-service mathematics teachers with the readiness to generate appropriate assessments and integrate critical thinking into their future teaching (Ismail et al., 2022). In fact, Paolucci (2015) pointed out that most pre-service mathematics teachers graduate from teacher training programs with a narrow understanding of critical thinking. This case impacts on the capability of them to utilize critical thinking in solving problems and formulating mathematical assessments.

The paucity of understanding of critical thinking, as identified by As'ari (2017), makes pre-service mathematics teachers in Indonesia often struggle to recognize the need for critical thinking applications in specific situations while solving problems. In addition, Rahaju et al. (2019) found errors in the modified mathematical problem content due to the absence of their critical thinking in checking the accuracy of data changes and considering the suitability of mathematical concepts. In addition, Rahaju et al. (2019) indicated how the absence of critical thinking in mathematical problem completion left pre-service mathematics teachers unable to utilize the devised solutions to evaluate the completeness and accuracy of the additional information. A narrow understanding of critical thinking also blinds pre-service mathematics teachers to recognizing the potential of content as a catalyst for practicing critical thinking (Altun & Yildirim, 2023). Hence, pre-service mathematics teachers need to assess

This study is part of a doctoral dissertation.

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Contribution to the literature

- The exploration of how pre-service mathematics teachers perceive the need for critical thinking in rich mathematical problem completion contributes to the existing literature on the construction of assessments following the necessary critical thinking criteria.
- The application of grounded theory in this research stands apart from the existing literature in the context of locating the trends of feedback in general and its consistency.
- The use of rich mathematical problems also differentiates from previous studies in invoking perceptions of critical thinking.

the extent to which critical thinking contribute to mathematical problem completion.

An appraisal of the extent to which pre-service mathematics teachers perceive critical thinking to contribute to mathematical problem solving can be derived from their perceptions. Pre-service mathematics teachers' perceptions depend on the extent of their comprehension and knowledge of critical thinking, which contribute to their own information processing and mindset formation (Lui & Bonner, 2016). The existing studies from several countries have captured how pre-service mathematics teachers conceptualize critical thinking. Ismail et al. (2019) sought the influence of pre-service mathematics teachers' perceptions in Malaysia on their readiness to integrate critical thinking instruction in class. In Jordan, Innabi and El Sheikh (2007) identified differences in how mathematics teachers perceive critical thinking before the reform era and after the reform era. Kusaeri and Aditomo (2019) explored the readiness of pre-service mathematics teachers in Indonesia to teach critical thinking in class.

While many existing studies have discussed perceptions of critical thinking in general, there is a lack of work exploring how pre-service mathematics teachers perceive the extent to which critical thinking contribute to mathematical problem completion. In consideration, Hokor (2022) found difficulties in solving rich mathematical problems due to the existence of ill-structured information and the completion needs certain approaches. It remains unclear how pre-service teachers perceive the extent to which rich mathematical problems require the contribution of critical thinking skills in their completion. This study seeks to address this gap by focusing on the following research question:

How do pre-service mathematics teachers perceive the extent to which rich mathematical problems require the contribution of critical thinking skills in their completion?

THEORETICAL FRAMEWORK

The following passage reviews prior research focusing on the pivotal role of individual perceptions, the conceptualization of critical thinking, critical thinking development through rich mathematical problems, and several approaches to engage critical thinking in practice.

The Pivotal Role of Critical Thinking Perception

Different perceptions of critical thinking across multidiscipline arise due to the absence of universal standards to conceptualize it. Critical thinking is described in philosophical terms as purposeful and goal-oriented thinking that strives to make judgments while meeting criteria for adequacy and accuracy (Bailin et al., 1999). Critical thinking psychologically centers on a set of cognitive skill and disposition elements that individuals demonstrate through their actions and behaviors (Lai, 2011). Unlike the philosophical and psychological perspectives, critical thinking educationally targets the information processing steps of analysis, evaluation, and creation (Bloom et al., 1956).

The existing literature conceptualized different versions of critical thinking perceptions. Individuals involve critical thinking in specific situations, particularly in solving conflicts or discussing a diversity of perspectives. This case indicates the existence of critical thinking as a temporary habit of mind (Innabi & El Sheikh, 2007). Another perception, on the other hand, points to the existence of critical thinking in all situations due to its constant involvement required by individuals. Despite the variations in conceptualization of critical thinking, the educational setting requires to develop a consistent consensus due to the implications of heterogeneity in teaching practices. Lack of clarity about the concept of critical thinking makes pre-service teachers often fail to translate their understanding into teaching practice, which challenge them to promote critical thinking and assess it due to their ignorance of when and how to involve critical thinking in practice (Johnson & Hamby, 2015).

Skilling et al. (2016) revealed that perceptions dictate how pre-service mathematics teachers structure content to engage critical thinking in practice. Pre-service mathematics teachers obtain significant implications for their professional development and conceptions of problem completion while engaging in critical thinking instruction (Jiang et al., 2022). Hence, perceptions of critical thinking skills and their contribution to mathematical problem completion are essential to the professional development of pre-service mathematics teachers to navigate their future teaching success.

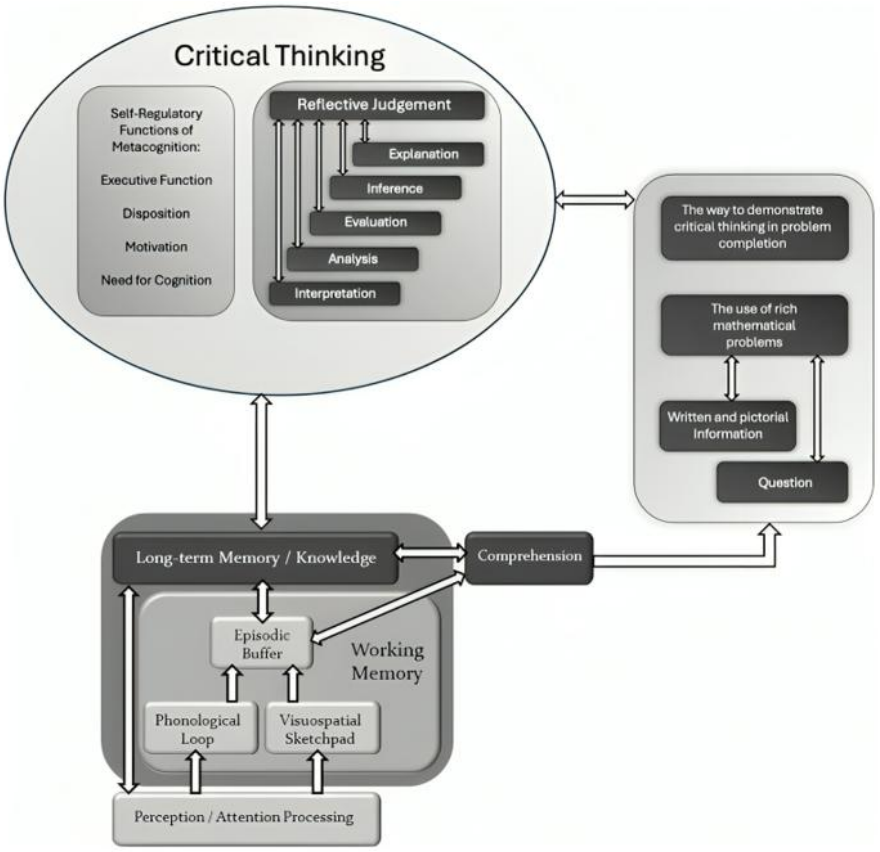

Figure 1. The integrative framework of critical framework (adapted from Dwyer et al., 2014).

This study incorporated additional components in the integrative framework of critical thinking to match the research focus, particularly the expansion of the comprehension part and connected it with critical thinking skill components including interpretation, analysis, evaluation, inference, explanation, and self-regulation (refer to **Figure 1**). We linked pre-service mathematics teachers' understanding of the use of rich mathematical problem content (e.g., written statements, pictures, and questions) and the application of approaches to engage thinking skills in their completion.

The Conceptualization of Critical Thinking Skills

Higher education institutions aim to elevate 21st century skills, with a particular focus on improving academic performance through the development of critical thinking. Although the partnership for 21st century skills recognizes that critical thinking is a crucial higher-order thinking, it still lacks a definitive conception of critical thinking. Abrami et al. (2015) reveal that critical thinking, as defined by the Association Delphi Committee, involves deliberate and self-regulated assessment through interpreting, analyzing, evaluating, and reasoning about problems utilizing available information. As noted by Ennis (2015) and Zhan (2021), critical thinking is a type of logical and reflective thinking that connects facts and theories with a coherent logical connection.

A few scholars expanded on the critical thinking conceptions via the lens of problem-solving. For instance, Bean (1996) contended that critical thinking is the act of identifying issues, finding necessary information, and making decisions to address problems. Moreover, critical thinking is defined as a way to solve problems and reach objectives by the application of diverse thinking approaches (Halpern, 2013). Hooks (2010) partitioned critical thinking into two phases. The initial step involves defining the problem and setting objectives, while the following step entails addressing the problem by locating pertinent information. Despite various definitions of critical thinking by researchers, all of them share the view that critical thinking is a cognitive approach affecting the learning process (Pintrich et al., 1993).

This study adopted the six components of critical thinking skills from the American Philosophical Association Delphi Project led by Facione (1990) and involving 46 experts from multidiscipline. This study draws on six components of critical thinking skills to characterize feedback from participants. All components of critical thinking skills and their descriptions are available in **Table 1**.

Table 1. The dimension of critical thinking skills (Facione, 1990)

Components	Expert's consensus description	Subskills
Interpretation	Comprehend and articulate the importance or meaning of diverse situations, data, or criteria	Classification, clarify meaning, & decipher importance
Analysis	Recognize actual connections among statements, questions, or concepts to reveal arguments, reasons, information, beliefs, or opinions	Examine ideas, identify arguments, & identify reasons and claims
Evaluation	Assess the trustworthiness of explanations regarding personal views and arguments; assess the coherence of logical connections between statements, descriptions, or inquiries	Question evidence, consider alternatives, & reach logically valid or reasonable inferences
Inference	Recognize and obtain the essential components to make logical conclusions; connect assumptions and theories; consider pertinent details and manage repercussions resulting from statements, evidence, judgments, concepts, descriptions, or inquiries	Assess credibility of claim & assess quality of arguments using inductive and deductive reasoning
Explanation	Present one's reasoning outcomes, defend them based on evidence, concepts, methods, criteria, and context, and articulate the reasoning through persuasive arguments	Reveal results, justify procedures, & provide arguments
Self-regulation	Monitor cognitive processes, materials used, and outcomes through self-awareness, particularly using analytical and evaluation skills to question, confirm, validate or correct own reasoning and inferences	Self-monitor & self-correct

The Development of Critical Thinking Skills Utilizing Rich Mathematical Problems

The provision of unstructured content in rich mathematical problems due to information overload or information limitation allows pre-service mathematics teachers to recognize the relevance of information to context. In addition, the exposure to challenging content in rich mathematical problems with real-world contexts enables pre-service mathematics teachers to perceive them differently depending on their personal perspectives (Cho & Kim, 2020). Their engagement in the process of meaning and definition calls for the application of critical thinking skills of 'interpretation'.

The incorporation of written statements and illustrations in the rich mathematical problem content navigates pre-service mathematics teachers to connect information across each other to provide evidence. Moreover, the diversity of information stimulates pre-service mathematics teachers to identify similarities and differences among several choices (Jaelani et al., 2023). These activities involve the use of critical thinking skill 'analysis'. The rich mathematical problems also provide pre-service mathematics teachers with opportunities to approach them in different ways. Pre-service mathematics teachers have the flexibility to organize how to complete problems effectively in their own way (Insorio & Librada, 2020). Their participation in the assessment and reflection on problem-solving effectiveness entails the application of critical thinking skills 'evaluation'.

The provision of pro and con questions in mathematical problems involves pre-service mathematics teachers in decision-making. The questions in the rich mathematical problem ask pre-service mathematics teachers to give statements as answers to the questions following evidence and consideration of various situations. The validity of their response hinges on the plausibility of their arguments and the precision of their solutions (Jaelani et al., 2023). The involvement of pre-service mathematics teachers in the decision-making process calls for the application of the critical thinking skill 'inference'.

Approaches For Involving Critical Thinking in Practice

The key to practicing critical thinking in mathematics stems from the application of suitable content and approaches. In Singapore, Li et al. (2020) adopted Socratic questioning to engage critical thinking in authentic task completion. Suggested questions such as what are other thoughts on this topic? what is the basis for this viewpoint? successfully led to a diversity of perspectives that further became a consideration before decision-making. Such questions facilitated a process of exploration and assessment of the benefits and drawbacks of every alternative. These activities involve critical thinking skills including analysis and evaluation. Furthermore, Li et al. (2020) used questions in authentic problem completion such as is there another meaning of the given statement? which information is needed? what needs to be resolved? These questions ask for definition variations and information characterization. All these activities need critical thinking skills to produce good interpretations. Additionally, the application of questions such as what do you conclude from the process? what makes the conclusion accurate? led to the provision of justification in decision-making. This activity involves critical thinking skills, specifically inference. Other scholars from Turkey adopted discussion for critical thinking in the completion of authentic tasks (Dolapcioglu & Doğanay, 2022). Discussion facilitated the examination of diverse

assumptions to derive the truth of the given context in the problem.

METHODOLOGY

The following part deals with the research design, participants, how to collect data, how to analyze data, trustworthiness, and ethical considerations.

Research Design

Our study adopted grounded theory to build a perception model of critical thinking skills, as it relies on the experience of participants, facilitates inductive analysis, and enables enhancement of understanding as data accumulate. Grounded theory operates on the premise of collecting data from a foundational level to develop a theoretical framework (Stough & Lee, 2021). We applied grounded theory with several considerations. The inductive nature of grounded theory drives us to construct theories that evolve from the data, rather than being constrained by existing theories or models. This approach greatly contributes to perception research, as it allows us to explore the different perspectives, experiences and behaviors of participants, particularly in relation to pre-service mathematics teachers. Furthermore, the iterative nature of data collection and analysis facilitated our improved comprehension of participants' perceptions as further data were obtained. This facilitates a refined and extensive comprehension of participants' perceptions, in which the focus is to capture how individuals perceive and rationalize their experiences.

Participants

The appropriate sample size in grounded theory research depends on the data itself rather than the set number of participants. A sufficient sample size is necessary to create an extensive data set for researchers to formulate theories according to the data. Sample size was determined by reaching theoretical saturation, which means that no more new data emerges about a category, categories were thoroughly examined, and connections among categories were confirmed (Corbin & Strauss, 2008). In grounded theory, having a sample size of 10-20 participants can be adequate, particularly when the goal is to develop a comprehensive understanding of a specific phenomenon (Lowe et al., 2020). Hence, a total of 19 pre-service mathematics teachers participated in the interviews, which met the criteria of grounded theory. They were enlisted through intentional and snowball sampling after receiving ethical approval. The criteria for selecting pre-service mathematics teachers to participate to this study included currently pursuing a mathematics education program, undertaking practice teaching in middle schools or participating in a microteaching program, and demonstrating acceptable oral communication performance.

Data Collection

This research took into consideration the adaptability and consistency of pre-service mathematics teachers in articulating their perceptions of the contribution of critical thinking skills to rich mathematical problem completion through a semi-structured interview (Galletta, 2013). We followed the framework as illustrated in **Figure 1** to develop questions for the interview protocol. Moreover, we adapted the interview protocol to the needs of this research through preliminary piloting with three pre-service teachers, we then made some changes to the phrasing, content expansion, and sequencing of questions (see **Appendix A**).

We embedded two types of questions in the final interview protocol: core questions and follow-up questions. We used core questions to reach general information on how pre-service mathematics teachers perceive the contribution of critical thinking skills to rich mathematical problem completion and follow-up questions to elicit more detailed data in response to their feedback. In this case, we asked for extensions of pre-service mathematics teachers' earlier statements with examples if we needed clarification on the perceptual alignment. Interview protocol was available in **Appendix A**. We conducted interview sessions with pre-service mathematics teachers for 60 minutes to 90 minutes. We also confirmed their personal details before the interview session.

This research initially engaged pre-service mathematics teachers to scrutinize the rich mathematics problem thoroughly (refer to **Appendix A**). Following this, we began to ask core questions about how the pre-service mathematics teachers perceived the necessary activities to do after receiving the problem. We then proposed follow-up questions regarding other necessary activities and asked for details of the mentioned activities. The next core question we ask concerned how pre-service mathematics teachers perceive the necessary activities specifically for processing information and which parts of the passage need their critical thinking to process. After receiving feedback, we asked further about how such information processing involved them in critical thinking. Afterwards, pre-service mathematics teachers received core questions about what activities were required to answer the question in the problem. We raised follow-up questions about how these activities need their critical thinking and what are the criteria for questions that need critical thinking in answering them. The last question we asked related to the ways in which they involved critical thinking in problem completion and what the process was.

Table 2. Marked label samples

Number	Marked Labels
1	I define what medium and large is.
2	I describe the terms spicy and original based on their appearance in the figure.
3	I group the components that should be in medium and large.
4	I determine the most affordable menu from diverse options through calculation.
5	I match the evidence to criteria such as budget sufficiency and individual preferences.
6	I think of different options like whole packs, whole individuals, or combinations of both.
…	…
216	Questioning encourages me to generate divergent thoughts and interpretations.
217	Questioning helps me to consider options and alternative ways to solve problems.
218	Discussion facilitates me to identify the emerging consequences of providing answers.
219	Through discussion, I can consider different perspectives in assessment.

Data Analysis

After the completion of the interviews, the recordings underwent verbatim transcription by NVivo 12 which served as a platform for data storage and management. We involved constant comparisons to analyze the data and carefully reviewed each interview transcript. Grounded theory was utilized to analyze the qualitative data, with the assessment followed by the steps of open coding, axial coding, and selective coding (Glaser & Strauss, 2017). Through a systematic analysis, we successfully identified main themes and patterns in the data, which contributed to our understanding of how pre-service mathematics teachers perceived the extent to which rich mathematical problems require the contribution of critical thinking in their completion.

Open coding

Open coding involved sorting and categorizing the raw empirical data. We adopted open coding to recognize primary concepts and categories linked to how pre-service mathematics teachers perceived the extent to which rich mathematical problems need the contribution of critical thinking in their completion. This process entailed the extraction of keywords found in primary sources, alongside the incorporation of native terminology from empirical sources to create unbound nodes representing the perceptions of them. Through this approach, we were able to construct a holistic understanding of essential themes and patterns in the data, leading to the formation of a theoretical framework for our investigation.

The first step in the open coding process involved labelling and conceptualization. We successfully recognized 219 labels from pre-service mathematics teachers' perceptions. The labels were then analyzed through a constant comparative method, allowing comparisons among them to uncover essential patterns and themes. The use of grounded theory in this process allowed us to streamline and enhance these labels, leading to the identification of 18 distinct concepts that encapsulated their perceptions of the contribution of critical thinking in rich mathematical problem completion (see **Table 2** and **Table 3**).

Table 3. Conceptualization results of marked labels

No	Conceptualization concepts	Marked label samples
1	Classification	I characterize what medium and large look like I group the components that should be in medium and large
2	Clarify definition	I define what medium and large is I describe the terms spicy and original based on their appearance in the picture I create my own definition of medium and large
3	Idea examination	The different preferences provoke me to think whether everyone can get the same menu and check budget adequacy through calculations to make the acquisition equality
4	Claim identification	I determine the most affordable menu from diverse options through calculation The difference in prices invites me to make claims about which menu is more affordable
5	Question evidence	I match the evidence to criteria such as budget sufficiency and individual preferences
6	Alternative consideration	I think of different options like whole packs, whole individuals, or combinations of both
7	Assess credibility of claim	I can select either agree or disagree, or both, depending on their assessment and consideration
8	The statement can be interpreted differently	The statement 'Rini treats eight of her friends to have lunch together' can have different meanings

Table 3 (Continued). Conceptualization results of marked labels

No	Conceptualization concepts	Marked label samples
9	The presence of multiple terms results in different perceptions	The terms such as medium and large enables me to perceive them differently
10	Matching the term usage with its appearance in figures	The figures help me build their perceptions of the different terms through the assessment of their appearance
11	Question elicits pre-selection considerations	Questions challenge me to select alternatives that meet criteria such as preferences, budget, and affordability
12	Questions invoke option diversity review	Questions lead me to check the effectiveness of different ways to obtain the best option
13	Questions lead to making free declaration	I can approve either agreement or disagreement based on their considerations
14	Questions allow the claim of two valid justifications	I can select both agreement and disagreement, as they are valid depending on which perspective I look at it from
15	Questioning denotes participation in the interpretation process	Questioning allows me to generate variation in interpretations and clarify definitions of term usage
16	Questioning shows participation in the analysis process	Questioning invites me to make claims and examine them through the provision of evidence
17	Discussions lead to informed judgment	Discussions facilitate me to find differences in the assessment of more affordable options, I will examine each option…
18	Discussions facilitate the reflection process	Discussions help me to reflect on their thinking from different views, particularly the consideration of several factors to show agreement or disagreement

Table 4. The result of concept categorization

Subsidiary categories	Concepts
Interpretation	Classification & clarify definition
Analysis	Idea examination & claim identification
Evaluation	Question evidence & alternative consideration
Inference	Assess credibility of claim
Interpretation variation through information processing	The statement can be interpreted differently & the presence of multiple terms results in different perceptions
Interpretation matching through information processing	Matching the term usage with its appearance in figures
Assessment through answering questions	Question elicits pre-selection considerations & questions invoke option diversity review
Decision-making through answering questions	Questions lead to making free declaration & questions allow the claim of two valid justifications
Questioning	Questioning denotes participation in the interpretation & questioning shows participation in the analysis process
Discussion	Discussions lead to informed judgment & discussions facilitate the reflection process

The following phase focused on the classification process using constant comparative analysis. We proceeded to compare and classify the identified 18 concepts. This facilitated the identification of 10 subsidiary categories that reflected the core patterns emerging from the data. These classifications provide greater insight into how pre-service mathematics teachers perceive critical thinking skills, leading to the creation of an overarching theoretical framework for our study (refer to **Table 4**).

Axial coding

Axial coding involved classification and comparative analysis of different categories or concepts. Axial coding focuses on dismantling and rearranging the unbound nodes of these perceptions to identify primary categories or concepts. This allowed us to uncover pivotal themes and patterns in the data, leading to the establishment of a more substantiated and elaborate theoretical framework for our study.

By constantly comparing and analyzing, we synthesized the ten subsidiary categories found during axial coding stage into four primary subcategories (refer to **Table 5**). Every subcategory featured various subsidiary categories extracted from the original classification. By defining these subcategories, we got a clearer and more targeted understanding of how pre-service mathematics teachers perceived the extent to which rich mathematical problems require the contribution of critical thinking in their completion, which contributed to the creation of a more elaborate and refined theoretical framework for our investigation.

Table 5. Results of the main axis coding for 10 categories

Sub-categories	Subsidiary categories
General perceptions of the contribution of critical thinking skills in rich mathematical problem completion	Interpretation
	Analysis
	Evaluation
	Inference
Perceptions of the contribution of critical thinking skills in information processing	Interpretation variation through information processing
	Interpretation matching through information processing
Perceptions of the contribution of critical thinking skills in answering questions	Assessment through answering questions
	Decision-making through answering questions
Perceptions of how to engage critical thinking skills in rich mathematical problem completion	Questioning
	Discussion

Selective coding

Selective coding involved the examination of relationships among primary categories or concepts and continual comparison to identify 'core categories' that could integrate various pre-service mathematics teachers' perceptions. By continuously examining 10 subsidiary categories and 4 subcategories, and by consistently comparing authentic data, we established a core category: 'pre-service mathematics teachers' perceptions of the contribution of critical thinking skills in rich mathematical problem completion'. This core category functioned as a vehicle for consolidating the emerging concepts, categories, and labels from the coding process, resulting in a comprehensive and integrated understanding of the research topic. The core category was instrumental in the development of a theoretical framework that identified primary patterns and themes in the data, providing a strong underpinning for the interpretation and analysis of our findings.

Trustworthiness

The concept of theoretical saturation, as informed by grounded theory, is fundamentally connected to the concept of trustworthiness (Glaser & Strauss, 2017). We performed a series of verification checks to secure the accuracy and credibility of our coding process. This included the execution of a coding consistency evaluation designed to reduce subjectivity and bolster confidence in the results of the coding process. We executed multiple coding iterations on the selected samples, allowing for a comparison with the results generated from the earlier coding. In addition, we evaluated theoretical saturation, drawing on the last five remaining interview profiles as our foundational source. This check enabled us to explore the potential identification of new classifications and connections emerging from these profiles and assess whether the theoretical model developed for this research had arrived at theoretical saturation.

Ethical Considerations

We adhered to the ethical guidelines for involving humans. The involvement of pre-service mathematics teachers in this study was voluntary and they had the right to withdraw consent for their participation at any time. We had kept their personal information including their names, affiliations, and contacts confidential.

FINDINGS AND DISCUSSION

The following section outlines the findings and discussion related to the four primary categories:

(1) general perceptions of the contribution of critical thinking skills in rich mathematical problem completion,

(2) perceptions of the contribution of critical thinking skills in information processing,

(3) perceptions of the contribution of critical thinking skills in answering questions, and

(4) perceptions of how to engage critical thinking skills in rich mathematical problem completion.

General Perceptions of Critical Thinking's Contribution to Problem Completion

Feedback on how 19 pre-service mathematics teachers perceived the contribution of critical thinking skills in solving rich mathematical problems is scattered on the network map (see **Figure 2**). Each different colored dot symbolizes a distinct concept. However, each dot is marked with a code that describes its corresponding category. We successfully identified four elements of critical thinking skills with reference to Facione (1990). They are analysis, interpretation, evaluation, and inference. For instance, the labelled code 'classification and clarify definition' relate to information processing, reflecting critical thinking skill component 'interpretation'. Likewise, the labelled code 'question evidence and alternative consideration' connect to eligibility review as per criteria, reflecting critical thinking skill 'evaluation'.

The network map illustrates that the dimension with the most participation belongs to 'alternatives consideration', while the dimension 'assess credibility of claims' contributes the least. Intriguingly, two pre-service mathematics teachers seem to recognize two dimensions of critical thinking skills.

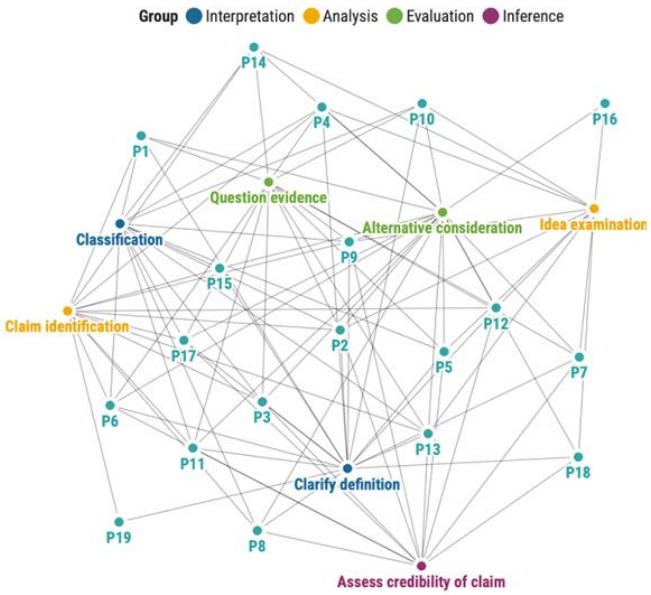

Figure 2. A network map indicating general perceptions of the contribution of critical thinking skills in rich mathematical problem completion (Source: Authors' own elaboration)

Pre-service mathematics teachers pointed out critical thinking practices in information processing. For example, in response to the use of multiple terms and pictorial presentations (see **Appendix B**), two pre-service mathematics teachers focused on how they interpreted some terms (such as medium and large) and their connection to the available figures.

> I characterize what medium and large sizes look like. For example, large contains larger portions particularly for fries/rice and drinks than medium … (P4).

> I create the definition of medium and large menu … Medium is the regular size, while large is the upgraded size menu … (P9).

As the quote shows, pre-service mathematics teachers highlight how they characterize and form their own definitions based on the information they receive. If we refer to Facione (1990), characterization falls under the sub-skill 'classification' and definition making leads to the sub-skill 'clarify meaning'. These two sub-skills involve their critical thinking in the process of information interpretation. Fonseca and Arezes (2017) stated that the involvement of critical thinking skills in the interpretation process dictates how individuals answer problems depending on what they perceive about the whole passage. As'ari et al. (2017) pointed out that the failure of pre-service mathematics teachers to answer problems correctly was due to the lack of critical thinking involvement in the process of how they perceived information, and they only relied on assumptions.

The application of critical thinking skills is crucial for pre-service teachers in navigating them to approach rich mathematical problems using appropriate ways. Pre-service teachers conceptualize critical thinking as the skill of probing relationships. For instance, two pre-service teachers highlighted how they dissect similarities and differences, especially in menu options and price lists (refer to **Appendix B**).

> Different preferences in menu selection provoke the thought of whether everyone can get the same menu. Rini's budget adequacy check through calculations to make the acquisition equality … (P2).

> The difference in package and single prices invites claims about which menu is more affordable. The unavailability of single menu prices in the list can be found through substitution of known variable values into the linear equation … (P15).

The quotes by pre-service teachers indicate their engagement in the examination of emerging thoughts due to gaps and the formation of claims for choice diversity. As suggested by Facione (1990), the examination of ideas and identification of claims are part of critical thinking skills application in the analysis process. Kusaeri and Aditomo (2019) found similar findings that pre-service mathematics teachers recognize the practice of critical thinking as learners involve information and evidence analysis in their perceptions.

Pre-service mathematics teachers perceived critical thinking as an essential skill in the whole process of assessment and evaluation to reach conclusions. For instance, in answering the suitability of Rini's menu selection (refer to **Appendix B**), two pre-service mathematics teachers suggested the need for critical thinking application in selecting the best alternative.

> I compare multiple menu pairs with each other and check each option whether it meets criteria such as budget adequacy and suitability to individual preferences … (P5).

> I can organize preference selection to figure out whether placing one order is cheaper than making two separate orders … (P11).

Pre-service mathematics teachers through their quotations pointed out critical thinking involvement in further examination of evidence conformity to criteria and selection of possible settings. Following the proposal of Facione (1990) on critical thinking skill characteristics, we categorize checking the suitability between evidence and criteria as the sub-skill 'question evidence' and recognize the selection of possible settings as the sub-skill 'consider alternatives'. These two skills contribute to the application of critical thinking in the evaluation process. As noted by Li et al. (2020), the use of critical

thinking skills in the evaluation process determines how individuals reflect on how they solve problems, especially to recognize shortcomings and how to overcome them.

Other views reflected situations where pre-service mathematics teachers were involved in critical thinking in inference formulation. In responding to the question related to the accuracy of Rini's menu selection (refer to **Appendix B**), two pre-service teachers elaborated on how their conclusions were made through multiple considerations related to budget sufficiency, suitability of individual preferences, affordability, and availability of option variations.

> I concur with the statement as Rini prefers to have a final cost that falls within her budget. The menu selection by Rini is not over her budget … (P1).

> I disagree with the consideration of equitable menu acquisition for everyone since two people obtain two chickens and one fries while the others are distinguished (P18).

> I can be in favor of agreement or disagreement depending on the assessment they adopt. The agreement came from the budget adequacy to pay Rini's order, while the disagreement was based on the consideration of other cheaper options (P6).

According to the excerpt shown, pre-service mathematics teachers focused on how they construct viable claims as answers to questions posed on rich mathematical problems. Their perceptions point to the need of critical thinking involving claim credibility assessment skills in the inferential process as noted by Facione (1990). As'ari et al. (2017) showed that the lack of pre-service mathematics teachers' involvement with critical thinking in problem-solving contributed to their inability to use inferential skills to provide complete arguments for their decision to answer as they did.

Pre-service mathematics teachers recognized at least two skill components. They are specialized in expressing the specific critical thinking skill components in which they master. As a result, pre-service mathematics teachers will teach in a way that focuses on developing the critical thinking they have mastered. In this context, there exists a discrepancy between their perceptions and knowledge attainment about critical thinking. Their inability to fully perceive the role of core components like analysis and evaluation will hinders their capability to construct a solid foundation for them to use their critical thinking skills in future teaching.

Perceptions of the Contribution of Critical Thinking Skills in Information Processing

The data emerged a second theme related to how pre-service teachers perceived written information as a stimulus to engage their critical thinking. The distribution of each pre-service mathematics teacher's responses for this theme can be seen in **Figure 3**. Their feedback on the availability of written statements and the use of terms prompted them to interpret and perceive them differently, we labelled these as 'interpretation variation through information processing'. Their responses on the availability of a combination of written statements and figures engaged them in the review process about the suitability and accuracy of both, we labelled these phrases as 'interpretation matching through information processing '.

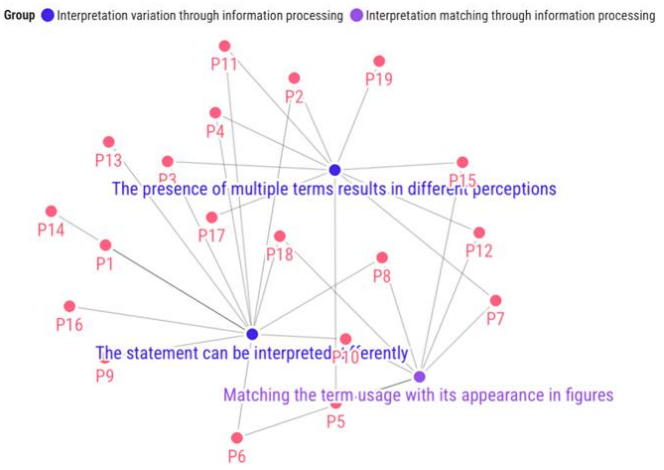

Figure 3. A network map showing perceptions of the contribution of critical thinking skills in information processing (Source: Authors' own elaboration)

The dimension with the largest proportion belongs to 'the statement can be interpreted differently', with 13 pre-service mathematics teachers contributing to this dimension. The dimension 'matching terms with their appearance in figures' received the least participation, with 8 pre-service mathematics teachers contributing to this dimension.

In this section, pre-service mathematics teachers demonstrated how specific reading passages provided opportunities for them to practice critical thinking skills. In the context of written statements (see **Appendix B**), two pre-service mathematics teachers pointed out that the statement relating to the number of people having lunch and the use of terms stimulated them to interpret it differently.

> The statement 'Rini treats eight of her friends to have lunch together' can have different meanings. For instance, A argues that only eight people have lunch, B contends that there are nine people since Rini participated, C mentions nine people due to the written information informing that Rini orders nine packages … (P3).

> Terms such as medium and large allow us to see them differently. For example, one described large

> as a larger menu than medium, while another specifically mentioned that large can upgrade rice, fries, and drinks from medium to larger size (P12).

According to the quote above, pre-service mathematics teachers indicated that their understanding depends on how they interpret and define the information they receive. As stated by Jaelani et al. (2023), ill-structured problems with some missing elements or unclear definitions make it difficult to understand, identify, and collect relevant information to solve the problem. Similarly, Cho and Kim (2020) found that difficulties arise in identifying and organising the necessary information due to the incompleteness of some elements and the broad scope of the problem.

Another group of pre-service mathematics teachers highlighted how the combination of figures and written statements engaged them in the practice of critical thinking skills. For example, one pre-service mathematics teacher stated how to build perceptions of different terms through the identification of their appearance in pictures.

> The figures help us build perceptions about different terms such as original and spicy through the assessment of their appearance such as shape, color, taste … (P8).

Pre-service mathematics teachers' feedback on the excerpt suggested that the provision of figures or illustrations clarified interpretations, located characteristics, and identified the accuracy of information. Li et al. (2020) presented similar findings with the use of illustrated fake news to engage critical thinking practices and the recognition of its importance. They stated the success of such stimulation to elicit a diversity of learner perspectives.

All pre-service mathematics teachers acknowledged the need for stimulation from good content to practice their critical thinking skills effectively. They characterized the content as a catalyst, however, especially for one skill component related to interpretation. They can go beyond this by concentrating on the content to incorporate additional components of critical thinking skills. Thus, pre-service teachers apparently have a limited understanding of how to utilize content to engage their critical thinking skills.

Perceptions of the Contribution of Critical Thinking Skills in Answering Questions

Further examination of the interview data enabled us to identify the third theme of how pre-service mathematics teachers perceived questions as stimuli to practice their critical thinking. In their feedback on stimulating questions to select and review effectiveness based on standards, we labelled these as 'assessment through answering questions'. Their feedback on stimulating questions to show disagreement or agreement with reasonable arguments, we labelled this phrase as 'decision-making through answering questions' (refer to **Figure 4**).

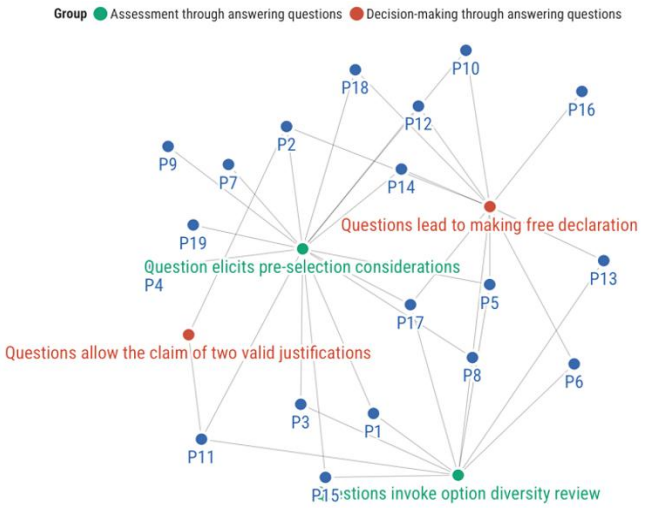

Figure 4. A network map revealing perceptions of the contribution of critical thinking skills in answering questions (Source: Authors' own elaboration)

The graph shows that 'questions allow the claim of two valid justifications' received the lowest contribution from two pre-service mathematics teachers and the dimension 'questions elicits pre-selection considerations' made the largest contribution.

In this part, pre-service mathematics teachers elaborated on how the questions involved them in the practice of critical thinking skills. In relation to the question of Rini's precision in selecting the menu (see **Appendix B**), two pre-service mathematics teachers pointed out the need for assessment of individual options and reflection on how to organize orders effectively.

> Questions challenge them to select alternatives that meet criteria such as suitability of individual preferences, budget adequacy, and affordability … (P2).

> Questions lead them to check the effectiveness of different ways to obtain the best option, whether one order generates a more affordable total price than two orders … (P15).

The quote above reflects the question works as a stimulation to involve them in the evaluation based on standards and criteria. Questions can provide opportunities for them to recognize the advantages and disadvantages of each option through consideration of various factors to arrive at the best choice.

Another group of pre-service mathematics teachers expressed different perceptions. In response to the question 'has Rini selected the right order?' (see **Appendix B**), three pre-service mathematics teachers

pointed out the need for inference to answer the question by sharing three different perspectives.

> I agree with Rini's decision because it suits her preference and does not exceed her budget … (P10).

> I express my disagreement by providing evidence and reasonable arguments about the existence of alternative of more affordable options … (P13).

> I choose both agree and disagree, as they are valid depending on which perspective we look at it from … (P11).

Pre-service mathematics teachers noted that the practice of critical thinking skills in approaching rich mathematical problems depends on the type of question. They further outlined the different thinking activities according to the type of question. For instance, questions starting with 'what' stimulate them to understand the content well and questions starting with 'how much' invite them in the use of formulae. Meanwhile, questions beginning with 'how' allow them to participate in the application of critical thinking skills due to the need to perform evaluation and assessment process. Similarly, questions starting with 'why' typically encourage them to reflect on their thoughts through the provision of logical arguments to corroborate the evidence. These perceptions reveal that the deeper the questions posed to them, the higher the quality of their thinking.

All pre-service mathematics teachers' responses pointed to the importance of accurate questioning to engage them in the practice of critical thinking skills. Pre-service mathematics teachers explicitly linked their feedback to specific components of critical thinking skills, as reflected in their perceptions. However, their perceptions are potentially expandable to include the other two components of critical thinking skills. Pre-service mathematics teachers need to give more specific explanations of situations involving critical thinking practices after they receive the questions.

Perceptions of How to Engage Critical Thinking in Mathematical Problem Completion

We mapped each category for the fourth theme with different colored points (refer to **Figure 5**). Each category represents feedback variance on how pre-service teachers perceive engaging critical thinking in rich mathematical problem completion. The network map shows the largest spread in the category of 'questioning'.

In this section, pre-service mathematics teachers shared different approaches to include their critical thinking in solving rich mathematical problems. Almost all pre-service teachers recognized questioning as the easiest way to show their critical thinking in practice.

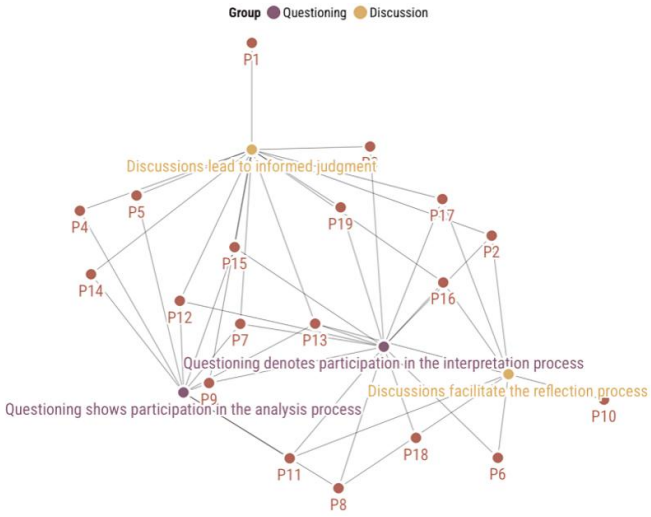

Figure 5. A network map showing perceptions of how to engage critical thinking skills in rich mathematical problem completion (Source: Authors' own elaboration)

For example, in responding to the availability of menu variants (see **Appendix B**), two pre-service teachers raised questions in the analysis process about how to organize effective orders, find alternative options, and provide evidence.

> What if Rini makes two orders? Is this more effective than one order? What is the evidence? How about the usage of coupons? (P14).

> Are there more affordable options? How do these options fit? Does the preference exceed Rini's budget? How can we claim that this is the least expensive option? (P5).

In another case, related to pictorial and written statements in rich mathematics problems (refer to **Appendix B**), two pre-service teachers raised questions in the interpretation process related to defining several terms and finding the true meaning of the statements.

> What are the components of medium and large? Do you find any differences? What do you mean by medium and large? How can they be categorized? (P7).

> Does Rini have lunch together or is it just her friends? How do you know if Rini will come for lunch? Why are the options for her two friends different? (P16).

The excerpts above indicate questioning as a catalyst for critical thinking skills in the interpretation and analysis process. This perception resonates with Li et al.'s (2020) study on the practice of critical thinking using authentic tasks and dialogue. They exposed the success of questioning in stimulating learners to identify relevant information, classify information, and assess information in several ways. In addition, questioning

enables learners to explore a diversity of perspectives in assessing the merits and demerits of various options.

Another group of pre-service mathematics teachers stated similar perceptions of asking questions to show their critical thinking skills in practice. Yet, two pre-service mathematics teachers modelled the application of questioning to gain foundational understanding of rich mathematical problem content.

> Who does the passage talk about? where does it happen? when does it happen? … (P1).

> How many people will have lunch together? who will have lunch together? what is the question? what are the menu options? (P10).

Their perceptions pointed to the lack of clarity in understanding critical thinking skill concepts due to the acquisition of basic understanding as their critical thinking standards. The inability of pre-service mathematics teachers to demonstrate their critical thinking through appropriate questioning was suggested by Er (2024) to be due to their lack of acquisition of subject and pedagogical knowledge.

Pre-service mathematics teachers shared about their participation in discussions on assessing rich mathematical problem content. In the context of the accuracy of Rini's menu selection (refer to **Appendix B**), two pre-service mathematics teachers pointed out the need for perspective diversity in the assessment and evaluation process.

> Discussions enable us to find distinctions in the valuation of more affordable options, we carefully examine each option including its advantages and disadvantages … (P17).

> Discussions direct us to reflect on thoughts from different perspectives, particularly the consideration of several factors to express agreement or disagreement … (P8).

As the quotation shows, the discussion facilitated them to engage in critical thinking through a process of assessment and reflection. This feedback resonates with the approach examined by Dolapcioglu and Doğanay (2022) that discussions provide learners with the opportunity to present alternative ideas and receive new knowledge from their peers. During discussion, learners present synthesized assumptions to verify their truth, they sought to convince each other of their perspectives and consider several aspects to achieve an accurate decision. In contrast, Innabi and El Sheikh (2007) mention discussions as a general way as collaborative learning can work in the classroom without requiring learners to actively engage in the critical thinking practice.

CONCLUSION

By pointing out how pre-service mathematics teachers perceive critical thinking skills, this research uncovered that each of them recognized the contribution of at least two elements of critical thinking skills in rich mathematical problem completion. Every pre-service mathematics teacher explained how critical thinking skills contribute to rich mathematical problem completion, particularly the use of basic skills such as interpretation and the use of core skills like analysis, evaluation, and inference. The descriptions about the role of each skill in critical thinking practices by pre-service teachers show that interpretation stimulates them to grasp the content comprehensively, analysis involves them to identify relations across information and provide evidence, evaluation leads them to reflect on their thoughts through the provision of reasonable arguments and careful consideration, and inference facilitates them to perform decision-making. Moreover, pre-service mathematics teachers suggested the need for content as a stimulus to involve them in critical thinking practices despite them only partially linking the application of skills to both content and questions. Another perception of pre-service mathematics teachers related to the way they engaged in critical thinking, this study revealed that active interaction in questioning and discussion stimulated them to put their critical thinking skills into practice in problem completion. This study notes that pre-service mathematics teachers' perceptions dictate how they organize experiences with critical thinking practices in the future.

Limitation

Regardless of the perceptions of pre-service mathematics teachers underlining how they need critical thinking in problem-solving, this single aspect calls for support from content knowledge to practice critical thinking effectively. This research proposes the need for pre-service mathematics teachers to demonstrate their sufficient knowledge and directly practice critical thinking skills in solving rich mathematical problems. Thus, we can comprehensively conceptualize how their perceptions and content knowledge contribute to critical thinking practices, particularly in what ways they demonstrate critical thinking skills in solving rich mathematical problems.

Implications and Recommendations

Our study shares with pre-service mathematics teachers' self-reflection on the extent to which their knowledge acquisition during the teacher preparation program significantly changed their perspective on critical thinking skills and their application in problem-solving. A snapshot of pre-service mathematics teachers' perceptions builds their future teaching paradigm on the need to engage students in critical thinking practices

while solving problems and the need for modelling in their applications. In addition, this reflection removes the negative stigma in the thoughts of pre-service mathematics teachers about the impossibility of involving students in critical thinking applications due to their unfamiliarity with how to incorporate critical thinking into teaching and difficulties in curriculum adjustment. In fact, pre-service mathematics teachers are stuck in their narrow thinking and close themselves to making changes to their teaching in the future.

Critical thinking skills practice needs mathematical content as a stimulus. Pre-service mathematics teachers' perceptions reflect how they evaluate the suitability of the content to engage their critical thinking skills in solving mathematical problems. Therefore, pre-service mathematics teachers need to use their insight to think further about the criteria in the content formulation to stimulate students' critical thinking. Moreover, teaching critical thinking skills through the provision of rich mathematical problems needs to incorporate modelling of how students do it. Pre-service mathematics teachers' perceptions indicate their assessment of the diversity of ways, including how they contribute to critical thinking practices. Reflecting on this, they can consider the strengths and weaknesses of each option, ultimately deciding to adopt a suitable approach for critical thinking practices in future teaching.

Future research may uncover how pre-service mathematics teachers can translate their perceptions of the role of critical thinking skills in problem-solving into direct classroom teaching practices. This study can serve as an evaluation for teacher preparation programs to equip pre-service mathematics teachers with adequate content knowledge and pedagogical knowledge, optimize their skills in organising students to engage in critical thinking practices, and enhance their own critical thinking skills.

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Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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APPENDIX A: INTERVIEW PROTOCOL

This study explored how pre-service mathematics teachers perceive the extent to which rich mathematical problems require the contribution of critical thinking skills in their completion.

Core Questions

1. What activities do you need to do after receiving this rich mathematical problem?
2. How does rich mathematical content engage you in critical thinking?
3. What activities do you engage in to process the given information?
4. Which parts of this passage are most likely to need your critical thinking to process?
5. How does written information engage you to think critically?
6. What activities do you undertake to answer the question in the problem?
7. How does the question in this problem engage you in critical thinking?
8. How do you engage critical thinking in this rich mathematical problem completion?

Advanced Questions

1. Are there any other activities besides the ones you have mentioned?
2. Could you please provide more details about the activities you have proposed?
3. How do the activities you have mentioned engage you in critical thinking?
4. Could you please elaborate on how the passages you mentioned engage your critical thinking?
5. How do these information processing activities engage you in critical thinking?
6. How do these question answering activities engage you in critical thinking?
7. Are there specific criteria for the types of questions that require critical thinking in answering?
8. Could you please share examples of questions that do and do not require critical thinking in answering?
9. Please explain further how such questions engage your critical thinking?
10. What specific situations involve you demonstrating critical thinking in this rich mathematical problem completion?
11. Could you please specifically model your critical thinking engagement through the approach you mentioned earlier?

APPENDIX B: A RICH MATHEMATICAL PROBLEM

Rini treats eight of her classmates to have lunch together at school to celebrate her birthday. However, two of her friends do not like spicy flavors and do not eat rice. Rini selects several menus for her friends in the McDelivery application, which provides an option of medium (M) or large (L) menus. The available menus are displayed in the figure below.

Chicken	○	Crispy	○	Spicy	○	Mix Only for All-chicken	
Upgrade	○	French Fries	○	Rice	○	Beverage	
Size	All-chicken, triple, and double with rice are only available in medium size						

All-chicken Package	M	Rp 119.000,00
Single Package	M	Rp 38.000,00
Single Package	L	Rp 45.500,00
Double with rice	M	Rp 55.000,00
Triple Package	M	Rp 90.000,00
Double with french fries	M	Rp 52.000,00
Double with french fries	L	Rp 59.500,00
Upgrade rice / french fries		Rp 5.000,00
Upgrade beverages		Rp 2.500,00

Voucher 1

Maximum 10% discount of IDR 50,000 with a minimum purchase of IDR 300,000

Voucher 2

Maximum 20% discount of IDR 80,000 with a minimum purchase of IDR 350,000

McDelivery application offers voucher one and voucher two as displayed in the figure above, which Rini can select when ordering. Rini can use one of the two available vouchers for one order. If Rini orders seven medium spicy single packages and two large crispy double with french fries with a budget of IDR 380,000, is this the right decision? Please explain your answer!

(Source: Authors' own elaboration)