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The cognitive gap in the mathematical thinking abilities of high school leavers for college: Are they ready?

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Abstract
After students spend approximately 12 years of formal math learning from high school, they bring a store of enormous "learned" mathematics factual knowledge to face the challenges and prepare for college/tertiary level learning. However, research has shown that early tertiary-level students struggle to learn college mathematics. The ability to think mathematically and use this learned factual knowledge (mathematical thinking) to solve higher-order thinking skills (HOTS) problems is essential to tertiary education. Thus, do these high school leavers have access to previously learned factual knowledge and use it effectively in solving these HOTS problems? This sequential research design study was conducted among 640 high school leavers who received an A in their national examination. In the first phase, the researchers investigated their mathematical thinking ability, followed by interviews with selected students on the difficulties and challenges they faced in solving the underlying problems. The findings showed that these students lack the ability to effectively use the previously learned factual knowledge from school mathematics to solve mathematical thinking problems. Secondly, they lack the habitual mind to check their answers after deriving a solution to a given problem. Thirdly, most rarely used heuristics to devise a strategy to solve fundamental math problems. Although the expectation of the school math curriculum over the last decade has been re-engineered towards "teaching students to think," this expectation has yet to be fulfilled. Thus, university educators must do more to guarantee that high school leavers can deconstruct their mathematical knowledge and reconnect it with the underpinnings and linkages of college mathematics requirement.

Keywords: mathematical thinking, school mathematics, higher-order thinking, heuristics, non-routine, problem-solving

INTRODUCTION

Various global education systems, including the Malaysian education system, have undergone significant reforms in the past decade to foster national development, specifically equipping citizens with the necessary skills to compete globally (Malaysian Education Blueprint, 2013-2025). The discourse surrounding curricular reform, particularly in primary and secondary education, has been prominent since the early 2000s. However, it is crucial to note that the majority of studies assessing mathematical thinking capabilities have predominantly concentrated on students of younger ages, typically around 11 to 16 years old, as evident in large-scale assessments such as trends in mathematics and science studies (TIMSS) and programme for international student assessment (PISA) (Heine et al., 2022; Perera & Asadullah, 2019; Wang et al., 2023).

Despite the widespread nature of these assessments, there has been a notable gap in research explicitly investigating the mathematical thinking abilities of high school leavers, particularly those on the brink of entering college. Previous studies have touched upon various aspects of educational reforms, yet the specific challenges faced by high school leavers in applying their mathematical knowledge to higher-order thinking skills (HOTS) problems remain inadequately explored (Demkir, 2022; Raflee & Halim, 2021; Raman, 2023). This gap is critical as it impedes a comprehensive

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Contribution to the literature

- The study addresses the lack of research on the mathematical thinking abilities of high school leavers transitioning to college, a critical period that has been overlooked in favor of younger students.
- The study reveals that high-achieving students struggle with higher-order mathematical thinking, indicating that current school curricula are inadequate for preparing students for college-level challenges.
- The findings imply the need for curriculum reforms that focus on teaching mathematical thinking and problem-solving, rather than just procedural knowledge.

understanding of whether the intellectual capacities of school leavers align with the expected cognitive demand at the tertiary level–a question of paramount importance in the ongoing context of curricular reforms.

There needs to be more study on the mathematical thinking abilities of high school leavers, which highlights the necessity of a comprehensive analysis of the cognitive gap during this critical transition. By using a sequential research design on 640 high school leavers who attained an A on their national exam, our study seeks to close this gap. By carefully selecting these high achievers, we can assess the quality of these students' thinking skills, who have shown to be highly proficient in mathematics knowledge. This selection will inadvertently enable us to examine the cognitive gap in mathematical thinking among academically inclined students. By concentrating on these students, we aim to gain insights into the challenges and obstacles faced by even the top-performing individuals during the critical transition from high school to college, providing a comprehensive understanding of the cognitive demands at the tertiary level.

LITERATURE REVIEW

Mathematics is a cognitive skill demanded for all levels of education, especially in today's rapidly changing world, particularly in terms of technological advancement, and the demand for this is unthinkable without mathematics (Hansson, 2015, 2020). Its significance is evident in its pervasive influence across various domains of life. Applying mathematical knowledge and skills to address real-life challenges in learning mathematics catalyzes students' holistic development (Osman et al., 2018). Problem-solving, recognized as the most effective approach for comprehending and applying mathematical concepts, facilitates contextualization and re-contextualization, enhancing the learning experience (Căprioară, 2015).

In mathematics education, problem-solving and mathematical thinking are intertwined, playing pivotal roles in fostering students' mathematical proficiency. The overarching objective of learning mathematics is to equip students to recognize and solve problems effectively. Research indicates that adept problem solvers demonstrate qualities such as testing and retesting ideas, making conjectures, self-awareness of their thinking processes, and the ability to generate and try new solution strategies (Bayat & Tarmizi, 2010; Parmjit et al., 2016; Schoenfeld, 1992). Students develop their ability to think mathematically as well as their ability to solve problems through these dynamic processes. Their HOTS are developed through problem-solving, which gives them the confidence to approach challenges effectively (Goodson et al., 2015). Looking at the many facets of mathematics education, it becomes clear that the relationship between problem-solving and mathematical thinking goes in tandem. Conceptual comprehension, logical reasoning, effective communication, and metacognition are all facets of mathematical thinking (Devlin, 2012; Mason et al., 2010; Schoenfeld, 1992;).

Mathematical thinking must be embraced as a comprehensive approach to mathematics education to develop competent problem solvers and proficient mathematical thinkers. Mathematical thinking entails breaking down math concepts into their structural and numerical roots and recognizing reasoning patterns (Devlin, 2012). Mathematical thinking is associated with developing proficient problem-solvers and mathematical thinkers by fostering an in-depth knowledge of concepts as well as successful problem-solving abilities. In this study, "mathematical thinking" is a pivotal focal point, emphasizing the cognitive skills essential for effective problem-solving and mathematical proficiency among high school leavers as they transition to college.

Nevertheless, it appears from recent studies (Chand et al., 2021; Faulkner et al., 2020; Kurnia et al., 2023) that educational establishments, to a large extent, fail to meet these demands sufficiently. These results highlight a deficiency in the ability of educational institutions to adapt to the evolving demands of learners. Faulkner et al. (2020) found that students' problem-solving abilities are hampered by an over-reliance on procedural information, which impedes cognitive development. In another study, Kurnia et al. (2023) found that 42% of year nine students in Indonesia showed poor statistical thinking, particularly when interpreting data, indicating a lack of statistical literacy among high school students.

A recent study in Pakistan shows that over 90% of primary students struggle with math and science (Arab News, 2022). This extensive research involved over 15,000 students from grades 5, 6, and 8 in 153 diverse public and private schools, revealing an average low test score of 27 out of 100 in mathematics. Fiji, a South Pacific nation, also grapples with significant concerns about mathematics achievement (Chand et al., 2021). These studies pinpointed an ineffective school mathematics curriculum and highlighted insufficient potential and competence among many teachers to teach mathematics effectively.

Many previous studies elucidate similar issues decades ago, but there is a lack of current findings on their status. In a study conducted by Lassila et al. (2009), prompted by concerns about the inadequacy of US education in producing enough scientists to meet future economic demands, it was revealed that high school leavers were ill-prepared for the cognitive demands of college-level education. Similarly, Scott (2016) found that students' lack of mathematical preparation before entering science classrooms hindered their ability to engage in meaningful learning. On the contrary, Scott (2016) also noted a significant increase in school test scores in math and science subjects. Another study by O'Brien and Dervarcis (2012) titled "Is high school tough enough?" found that approximately 40% of high school leavers were unprepared for entry-level employment or college courses. They advocated for a more rigorous curriculum to better equip high school leavers to tackle the cognitive challenges of higher education. Similarly, Shaughnessy (2011), a former President of the National Council of Teachers of Mathematics, highlighted issues related to high school students' readiness for tertiary education. Notably, these studies conducted about a decade ago have not been followed up with subsequent research findings based on a literature searchThese issues of concern, as mentioned above, are also prevalent in the Malaysian context of mathematics learning. Various evidence has been provided in the local literature on the low intellectual mathematics knowledge of high school leaver. In the study over the decade by researchers (Aida, 2015; Parmjit & White, 2006; Parmjit et al., 2016; Roselainy et al., 2013), they have concluded that these school leavers' intellectual capacity does not match with the expected level of cognitive demand at tertiary level. According to the findings of Parmjit and White (2006), the grades gained in the national examination (SPM) exams do not relate to their higher-order thinking abilities. Similarly, Roselainy et al. (2013) echoed a proposal to enhance math pedagogical practices in STEM education to make them more relevant and meaningful in a way that could further develop students' capabilities. Thus, action is warranted to curb these concerns, notably in the context of learners' cognitive growth in mathematical thinking.

Mathematics is one of the "micro filters" regulating entry into tertiary education, especially in STEM education. The current model of pedagogical practices in schools is outdated (Parmjit et al., 2016; Schoenfeld, 1992; Shaugnessy, 2011). At its micro level, do the various topics of math courses learned in high schools, such as calculus, algebra, trigonometry, geometry, and statistics, cater to the HOTS demanded at the tertiary level? The new curriculum seeks to develop learners "who can think mathematically and who can apply mathematical knowledge effectively and responsibly in solving issues and making decisions" (Malaysian Education Blueprint, 2013-2025, p. 2). The phrase "to think mathematically" was incorporated in the statement of objectives for the secondary school mathematics curriculum to emphasize the significance of mathematical thinking among high school students. Devlin (2012) asserts that mathematical thinking is a way to learn a math concept by breaking it apart and analyzing it until learners find its numerical and structural roots and ways of Thinking. It is a dynamic process that helps learners understand complex structures by compiling what they have already learned (Mason et al., 2010). The problem must be challenging, engaging, and within the learners' proximal development zone to develop their thinking. Mathematical thinking occurs when tertiary-level problem-solving requires high-level thinking skills. Schoenfeld (1992) argues that a curriculum that teaches only mathematical facts and methods is no longer valid.

Issues such as the underperformance of students in international math studies like TIMSS and PISA, challenges in STEM education, particularly low enrollment, and incongruence between high school leavers' intellectual capacity and the cognitive demands of tertiary education prompted the introduction of new curricula under the Malaysian Education Blueprint (2013-2025). These changes aim to address these multifaceted challenges comprehensively. This new curriculum's thrust emphasizes students' critical thinking, creative thinking, and problem-solving abilities. It targets being in the top third of nations by 2025, despite the country's history of consistently being in the bottom third in PISA and TIMSS. What impact have these reforms had on the tertiary level since its inception in 2014? Does the new math curriculum prepare high school leavers well enough for college-level cognitive readiness?

In this study, the indicators of students' cognitive disposition and ability to solve mathematical thinking problems draw inspiration from Devlin's (2012) "Introduction to mathematical thinking." Devlin's (2012) work underscores the importance of developing general thinking abilities beyond the rote application of formulas, aligning with the broader goal of cultivating mathematical thinking. The assessment approach in this study resonates with the literature on mathematical thinking, where non-routine problems are considered essential for fostering a deeper understanding of mathematical concepts (English & Kirshner, 2015; Hughes et al., 2006). These studies highlight the significance of problems that require a flexible application of fundamental concepts, reflecting the general thinking ability required for mathematical thinking. Addressing the conceptual challenges in understanding mathematical thinking problems involves primarily drawing upon general thinking ability. Nevertheless, these problems necessitate specific thinking abilities, including critical thinking and logical reasoning, within the broader context of general cognitive disposition (Devlin, 2012; Mason et al., 2010).

Thus, it is necessary to evaluate students' learning to understand the present influence of instructional methods on their educational progress, particularly concerning the readiness of high school leavers to confront the complexities of the tertiary-level mathematics curriculum. The assessment process is inevitable regarding instruction simply because it helps navigate the overall experience and works as a check and balance in ensuring educational goals are duly met. Through assessment, questioning takes place, and it forces one to think. For example, "Does the content taught to students in the classroom commensurate with what we think is being taught?" and "What are students supposed to be learning, and are they learning so accordingly?"

Thus, this research evaluated the cognitive readiness of high school leavers for the demands of tertiary-level education, examining their progression in mathematical thinking. The research questions posited for this study are, as follows:

1. What is the extent of the student's cognitive disposition ability in solving mathematical thinking problems?
2. What are the conceptual challenges in understanding mathematical thinking problems?

METHODOLOGY

This study employs a mixed-methods approach, utilizing a sequential research design, to comprehensively investigate the cognitive gap in mathematical thinking among high school leavers. The quantitative phase of the study involved a descriptive design, aligning with Kothari's (2004) definition as a method that "describes, records, and interprets phenomena without manipulation of variables that either exist or previously existed" (p. 120). This phase entailed administering a paper-and-pencil test, the mathematical thinking test (MTT), to 640 high school leavers ages 17 to 18. The data collected from this phase offers insights into students' mathematical thinking development over eleven years, from primary to secondary levels of mathematics education.

In the second phase, interviews were conducted with six purposefully selected students to capture their progression in mathematical thinking. Utilizing interviews proved crucial in uncovering students' difficulties, challenges, and misconceptions about learning concepts, aligning with Merrifield and Pearn's (1999) recognition of its effectiveness in assessing learners' mathematical thinking development. The selection criteria of the students were based on students' performance on the MTT scores, including two high achievers, two intermediate achievers, and two low achievers. This approach facilitated identifying distinct thought processes, stumbling blocks, and difficulties faced by each group of students.

While the study leans towards a qualitative emphasis, the deliberate inclusion of the quantitative component with 640 samples ensures a robust and detailed understanding of the cognitive gap phenomenon, leveraging both quantitative foundations and qualitative depth in exploring the research question. The foundational quantitative analysis establishes a crucial baseline before delving into qualitative interviews, offering a quantitative overview that complements subsequent qualitative insights.

For the quantitative approach, the MTT instrument developed by Parmjit et al. (2016) was adapted for this study to assess the mathematical reasoning proficiency of high school leavers. This study focuses on evaluating mathematical thinking in high school leavers. Unlike broader tests such as Watson-Glaser and California critical thinking skills, we opted for Parmjit et al.'s (2016) instrument because it emphasizes foundational mathematical reasoning. This intentional selection ensures a detailed examination of mathematical thinking abilities in our study.

The primary focus of the test centered around non-routine problems, which aim to enhance students' mathematical reasoning abilities and instill the notion that mathematics is a creative endeavor (Polya, 1973). This test had ten questions from school math that covered the fundamentals of ratio and proportion, algebra, basic permutation and combination, sequence, indices, simultaneous equations, and fundamentals of numbers. All the questions were classified as non-routine, meaning that no formulas were required to be remembered, and the employment of calculators was not allowed. Examples of the questions are as follows:

- *Three hoses fill a pool. The first hose fills the pool in 3 hours, the second in 4 hours, and the third in 12 hours. How long will it take to fill the pool with all three hoses open?*
- *There are seven students in the meeting room. Each student shakes hands with each other except for themselves. How many handshakes are made altogether?*
- *Find the last digit of 32007.*
- *A book's pages are numbered with 993 digits by a printer. How many pages does the book have?*
- *What is the digit in the one's position of the total after the first 97 whole numbers are added up? 1 + 2 + 3 + 4 + . . . + 94 + 95 + 96 + 97.*

Table 1. Paired sample statistic

Test		Mean	N	SD	t	df	Sig.
MTT	Test	8.97	33	3.68	-1.437	32	0.160
	Re-test	9.10	33	3.60			

Note. Maximum score: 33

Table 2. Test-re-test paired samples correlations

Test	N	Correlation	Sig.
MTT test-re-test	33	0.991	0.00

All the selected questions demand a flexible application of fundamental concepts, aligning with the general thinking ability required for mathematical thinking. This approach is consistent with Devlin's (2012) perspective, which emphasizes breaking down mathematical concepts into their structural and numerical roots while recognizing reasoning patterns.

The validation of the MTT entailed the engagement of seven experts in the selection process. The content validity index (CVI) and content validity ratio (CVR; Lawshe test) were employed to ascertain content validity. The MTT test items exhibited strong validity, with scores ranging from 0.875 to 1.000, indicating their appropriateness for assessing students' mathematical thinking. The comprehensive evaluation yielded an impressive overall S-CVI of 0.957 (Polit & Beck, 2006; Waltz et al., 2005; Zamanzadeh et al., 2014) for the 10-item scale. In tandem, the CVR values, ranging from 0.714 to 1.000, signified unanimous expert consensus, categorizing each item as "essential" for the instruments (Ayre & Scally, 2014).

The reliability of the MTT was assessed through a test-retest reliability analysis. This analysis aimed to ensure consistency in the test results before and after a one-week interval for the same individuals. Administered to 33 students, the findings, as presented in **Table 1** and **Table 2**, reveal a robust and statistically significant correlation ($r = .991$, $p < .05$) in the test-retest reliability, indicating a high level of consistency. Consequently, considering both validity and reliability analyses, the MTT instrument demonstrates its effectiveness as a valid tool for measuring students' proficiency in mathematical thinking.

The results from the validity and reliability tests mentioned above affirm that the MTT is valid for measuring students' mathematical thinking ability.

Furthermore, this was not a speed test; students were given one hour and fifteen minutes to answer the questions. This study examines students' conceptions of mathematics; thus, the working steps and procedures were considered when assigning the marks based on a pre-set criterion. Each question was assigned three points, yielding a maximum score of 30 on the MTT (**Table 3**).

For the qualitative research design, interviews were conducted with the selected students. The primary interview questions were from the MTT, followed by probing questions aiming to elicit the thought processes used by students in assessing the conceptual difficulties they faced in solving the given problems. In these interview sessions, the researchers got an opportunity to study the causes of each step and which heuristic was employed by the respondents. These complete transcripts were necessary to accurately represent what students had to say and to serve as a source for the long quotes often included in qualitative research reports as part of the interpretation validation process (Shenton, 2004). Verbatim transcripts strengthen a study's "audit trail" by providing more evidence (Sacks, 1984, p. 21).

RESULTS

The first section presented the study findings from the paper and pencil test administration comprising ten questions and the item analysis for each question. This was followed with interviews with students in the next section. We assess their non-routine problem-solving skills, the difficulty they face, their content knowledge, and their ability to use it to solve the given problems.

Ability to Solving Non-Routine Problems

Research question. What is the extent of the student's cognitive disposition ability in solving mathematical thinking problems?

The data in **Table 4** reveals that the scores achieved by 640 students engaged in the research are a low 9.15 (standard deviation [SD] = 3.84). In other words, these students attained a low score of 30.5% ([9.15 × 100]/30) on the MTT.

Item Analysis of Mathematical Thinking Test

This section analyses the ten questions in phase one of the MTT. **Table 5** indicates students' challenges on the test, which gives background information on high school leavers' conceptual comprehension and stumbling blocks of basic math concepts.

Table 3. The scoring rubric

Score	Description
0	No effort was made; this was a failed attempt.
1	Some aspects of problem are identified, but solutions that address those aspects are either insufficient or unsuitable.
2	Determine most aspects of the problem and provide at least one viable solution despite certain flaws.
3	Determine all components of the problem; the suitable strategies are presented along with the correct response.

Table 4. Mathematical thinking test scores

Test	N	Mean	SD
MTT scores	640	9.15	3.84

Note. Maximum score: 30

Table 5. Item analysis of MTT

Question	Correct (%)	Incorrect (%)
1	53.3	47.7
2	26.6	73.4
3	24.8	75.2
4	28.4	71.6
5	13.3	86.7
6	45.5	55.5
7	15.9	84.1
8	19.3	80.7
9	27.7	72.3
10	34.2	65.8

Table 5 shows that the questions students faced difficulty with based on the 50% or more incorrect responses were all the questions except question 1. The findings depict that students had difficulty solving problems requiring higher-order thinking skills. These non-routine problems elicit students' mathematical thinking skills. However, based on their national examination results, these students were considered high achievers in mathematics, and they still lack the cognitive repertoire one expects to have. This outcome is consistent with previous findings from both local and international contexts over the last decade (Adams, 2014; Aida, 2015; Intan, 2016; Parmjit et al., 2018). The findings suggest that, despite graduating from high school, most students lack the cognitive skills and growth required to meet college's academic requirements. Parmjit et al. (2018) viewed this downfall due to the familiar proverb "practice makes perfect". This proverb might be true for mastery skills for arithmetic operations but not for developing mathematical thinking. Students "practice" these skills to get the correct answer. In other words, they neglect context, structure, and conditions, and students do not produce the "richly interconnected spaces." Cooper (1988) identifies as necessary for building mathematical thinking. They end up with islands of superficial knowledge without a boat to travel from one end to the other.

The following section's findings from the interviews detail the difficulties encountered in cognizing the mathematical thinking problems that greatly hindered students' mathematical thinking development.

Difficulties Faced by Students

This section discussed samples of students' incorrect responses to the MTT. These incorrect responses were then probed to investigate the root of these difficulties faced via interviews. Due to space constraints, six interview participants were selected for this paper to determine their mistakes and difficulties in solving the problems. The selection criteria were grounded on students' performance on the MTT, including two high achievers, two intermediate achievers, and two low achievers. This criteria enabled, if any, the identification of distinct thought processes, stumbling blocks, and difficulties faced by each group of students. Coding for the interviews was used to identify the respondents according to respondent number and achievement. The coding was as follows:

- Students number: 1 to 6 and
- Achievement: L: low, I: intermediate, and H: high

Notification for each participant:

- Student 1, low achiever as S_{1L}
- Student 2, low achiever as S_{2L}
- Student 3, intermediate achiever as S_{3I}
- Student 4, intermediate achiever as S_{4I}
- Student 5, high achiever as S_{5H}
- Student 6, male, high achiever as S_{6H}

Did students lack factual knowledge or access it poorly?

From the interviews, all the respondents did not face problems in understanding the problems that, to an extent, asserted that the math knowledge required for each question was within their zone of proximal development.

> S1L: I understand the questions quite easily, but I don't know what concept to use … how to answer the question."

> S2L: This question seems easy but challenging because … I am not sure which math formula or concept to use.

> S3I: I am not sure how to make a connection … which concept and formula to use.

> S4I: The problems given are interesting … I like it … It seems easy but difficult to solve.

> S5H: These problems seem easy … but are definitely challenging when I try to solve them because, quite often, I am not sure what fact to look for … in fact, I get so confused about how to solve the problem ...

These statements above elucidate the fundamental descriptors of non-routine problems, such as Kantowski (1977), "an individual is faced with a problem when he encounters a question he cannot answer or a situation he is unable to resolve using the knowledge immediately available to him (p. 163). Similarly, Woodward et al. (2012) highlighted these non-routine problems that cannot be addressed with a known approach or

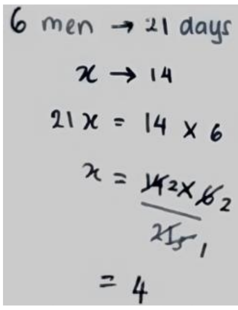

Figure 1. S_{1L} incorrect response for item 1 (Source: Field study)

memorized formulae that demand analysis and synthesis with the aid of critical thinking.

Factor 1: Lack of a habitual mind in checking their answers

The first item is a ratio and proportion item, which is widely used in the literature. This item aims to assess students' ability to use proportionality in solving problems. The quantitative analysis revealed that 47.7% (n = 306) of the 640 students attained an incorrect solution to this problem. A further probe indicates that 92.7% (n = 284) of these students obtained an incorrect response of four instead of nine as the correct solution to this problem.

Two factors embedded in this problem inhibited students from finding the solution. The first factor relates to not looking back to check their attained solution, and the second relates to the rote application of the formulaic cross multiplication method. The following is the verbatim that took place between the researcher and the participating student.

Question 1. If it takes six men 21 days to paint a house, how many men will be needed to do the same job in 14 days?

> R: Do you understand the question and related to which topic?
>
> S1L: Yes, it is related to proportion.
>
> R: Please solve this item.
>
> S1L: [After about 2 minutes, he responded] Four.
>
> R: Four what?
>
> S1L: [Hesitated for a while] … err … four days … [hesitated again] … no … four men. Yes, four men.
>
> R: How did you solve it? [Showing me his procedures (refer to **Figure 1**)].
>
> R: Can you please explain?
>
> S1L: Six men takes twenty-one days, so x men will take [pointing at his steps] fourteen … So cross multiply … twenty-one x equals fourteen times six … x is four!

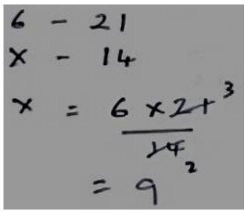

Figure 2. S_{3I} correct response for item 1 (Source: Field study)

S_{1L} utilized a mechanical procedure called cross multiplication, commonly used in schools, to solve the problem. This cross-multiplication refers to a process where the numerator of the first fraction is multiplied by the reciprocal of the second fraction's denominator and vice versa, setting the products equal.

All the respondents produce "4 men" as the answer during the interviews. Scaffolding was introduced to guide and probe students' thinking.

> R: If one needs six men to paint a house in twenty-one days, please look at the question. Will you require more or fewer men to paint it in a shorter time of fourteen days?

S3I began to ponder and was perturbed based on his facial expression.

> S3I: Something is not correct because … you definitely need more men!
>
> R: Why?
>
> S3I: Because if six men can paint in twenty-one days, then definitely more men are needed for fewer days, err … fourteen days.
>
> R: So, where is the mistake?
>
> S3I: This should be an inverse proportion.
>
> R: What do you mean by inverse proportion?
>
> S3I: More men fewer days or fewer men more days.
>
> R: So, what is the answer? Can you do it mentally?
>
> S3I: The answer will be twenty-one per (over) fourteen times six … three over two times six and … nine men … Let me check my answer.

The procedures in **Figure 2** were used.

> R: You were very sure of the answer as four men earlier. Why was that?
>
> S3I: This was a direct question we always do … I should have checked my answer if it makes sense!

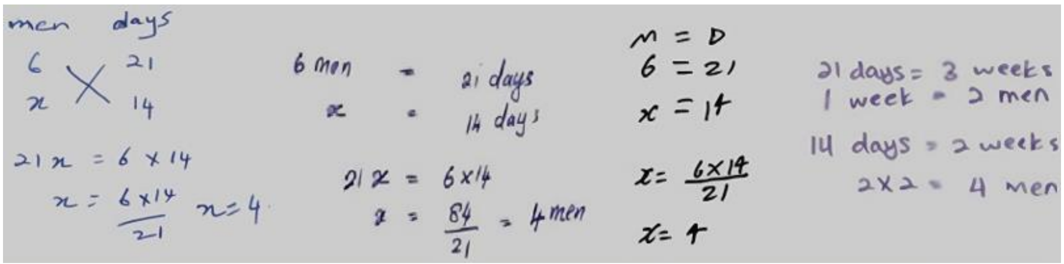

Figure 3. Samples of incorrect responses for item 1 from the paper and pencil test (Source: Field study)

Most students (based on the paper and pencil test script) utilized this cross multiply method to attain an incorrect solution of 4. The findings indicate poor algorithm operations (14 × 6)/21, indicating students' superficial comprehension of proportion and ratio. Students failing to double-check their answers to see if they make sense is a massive cause for concern. According to the data, most students answered "four guys" since they did not comprehend that the question was about inverse proportions. **Figure 3** illustrates samples of the incorrect solution obtained in the paper and pencil test among the students involved in the study.

Polya (1971) asserted that "looking back" when the problem has been solved maximizes learning opportunities. By re-examining the result and the route that led to it, students may solidify their information and improve their problem-solving skills. Instilling the habit of looking back extends beyond confirming answers and the procedures used to achieve answers, as it maximizes problem-solving learning opportunities.

Factor 2 and factor 3: Inability to relate with formulaic structure learned in school and lack of a heuristics repertoire in solving problems

The second and third factors that students faced were their inability to relate and apply the various formulas learned in school and a lack of heuristics repertoire to solve the non-routine problems. The following problem exacerbated this factor.

Question 8. There are seven students in the meeting room. Each student shakes hands with each other except for themselves. How many handshakes are made altogether?

One would expect the following procedures commonly learned in school (the topic of combination and permutations) to be utilized to solve the problem:

$$^7C_2 = \frac{7\times 6}{3} = 21. \qquad (1)$$

The paper and pencil test findings revealed that 80.7% of the 640 samples involved in the study responded incorrectly to this problem. Within these responses, approximately 82% (n = 423) left a blank space without attempting to solve it. None of the 640 samples involved in the study could use this learned combination formula to solve the problem. Further analysis from the paper and pencil test suggests that 9.1% (n = 58) of the 640 sampled students attempted to use heuristics to attain the solution. Examples of the heuristics used are shown in **Figure 4**.

The interviews suggest that students could be guided to solve the problem with scaffolding. S$_{4I}$ could not solve the problem; however, it reaps the benefits of scaffolding.

> R: Can you solve the problem?
>
> S4I: No, challenging to solve.
>
> R: Have you learned or solved this type of problem in school?
>
> S4I: No, I don't think so.
>
> R: Let me give you a hint. Say you have two students, A and B. With two students, how many handshakes?
>
> S4I: Two students … one handshake.
>
> R: Three students? [mumbled two get one, three get …].
>
> S4I: Three handshakes.
>
> R: What about four students?
>
> S4I: I think I know how to solve the problem …

S4I started working on the sheet of paper. After working for about 4 minutes.

> S4I: Twenty-one handshakes are the answer [pointing to his heuristic as shown in **Figure 5**] for seven students.
>
> R: This is for five students; the question is for seven students.
>
> S4I: You see, there is a pattern one, three, six, ten, then will be (heard saying five) fifteen and then (heard saying six) twenty-one.
>
> R: Tell me more about this pattern.

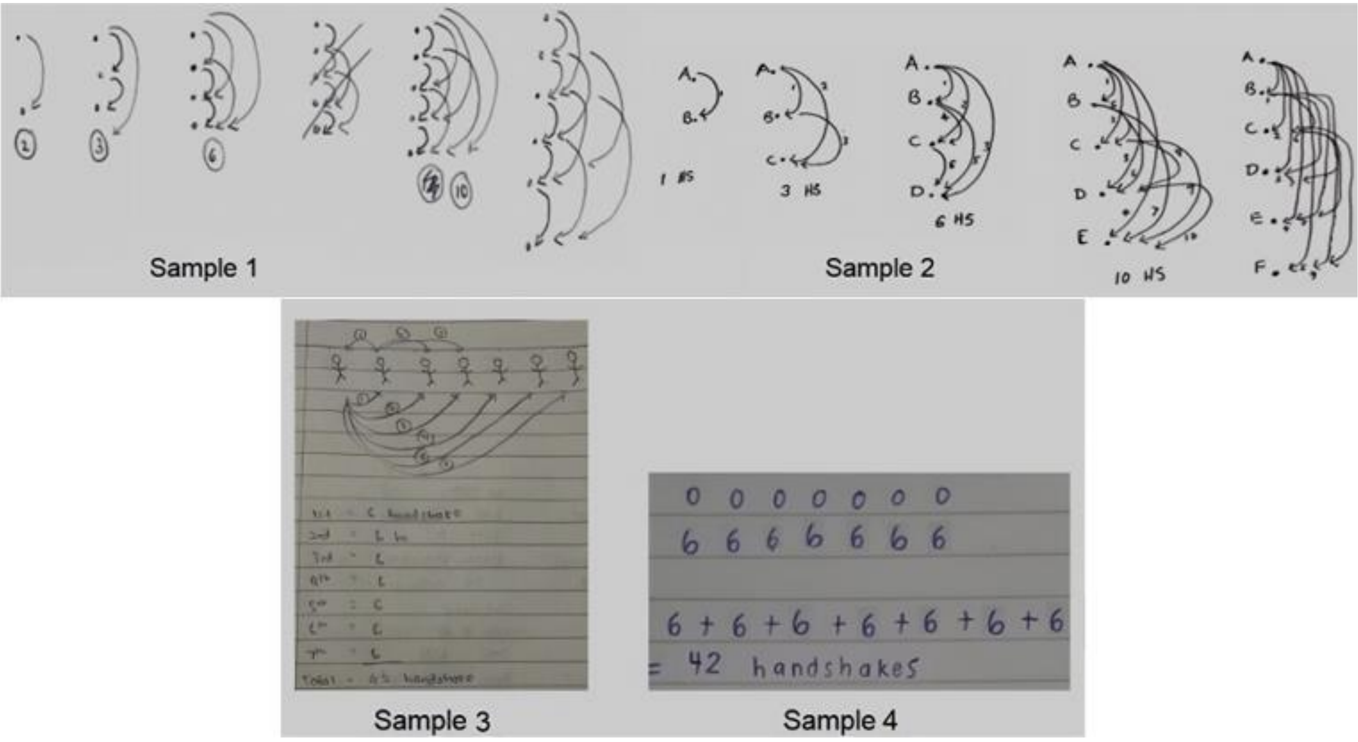

Figure 4. Samples of respondent's usage of heuristics in the paper and pencil test (Source: Field study)

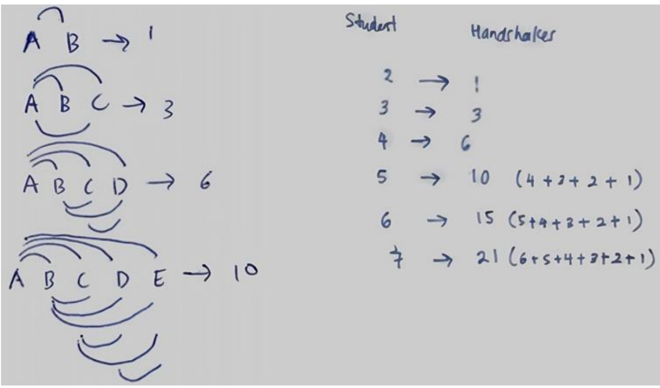

Figure 5. Heuristics for question 8 by S_{4I} (Source: Field study)

S4I: You see, from one to three, you add two, then three to six, add three, add four, five and then six.

With scaffolding, S_{4I} solve the problem by using the drawing heuristics and then recognizes a pattern to provide the solution of twenty-one handshakes.

S_{5H} was also successful in solving the problem using a pattern recognition heuristic.

R: How many handshakes are made altogether with seven students?

S5H: Twenty-one.

R: Please explain.

S5H: There is a pattern here (pointing to his systematic list-refer **Figure 6**): two students, one handshake, three students, three, four students, six and so on … seven students, you get twenty-one.

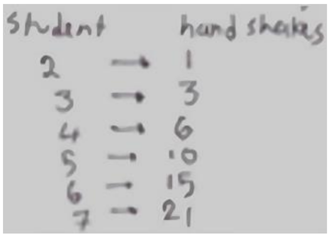

Figure 6. Heuristics for question 8 by S_{5H} (Source: Field study)

R: What about ten students? How many handshakes?

S5H: Nine plus eight plus seven plus six plus five plus four plus three plus two plus one … will be the answer!

S_{5H} made a systematic list and discovered a pattern in deriving the solution.

Common Challenges in Mathematical Thinking Across Achiever Groups

Specific challenges emerged across achiever groups when examining students' mathematical thinking abilities. A prevalent issue was the struggle to effectively apply factual knowledge, with students facing difficulty identifying the relevant mathematical concepts and formulas. Evidently, this challenge was encountered among all the low, intermediate and high achievers. Furthermore, a prevalent challenge among each group was the lack of a habitual thought process when verifying answers. Even high achievers showed a tendency to forget this vital step. A lack of a repertoire of heuristics for non-routine problem-solving and a failure to relate acquired formulaic structures to issues were other underlying characteristics that affected all groups. In order to address these issues, scaffolding interventions were helpful, emphasizing the need for focused techniques to improve mathematical thinking abilities. Developing a more comprehensive grasp of mathematical ideas and promoting habitual checking is essential for efficient problem-solving, regardless of achievement level.

The interviews and paper-and-pen test results revealed why students could not apply the previously learnt formulaic structures to solve the presented problem. On the other hand, high achievers used heuristics by looking for patterns in their attempts to solve problems. According to Parmjit et al. (2016), students demonstrate weaknesses in their cognitive thinking skills and mathematical aptitude even though their mathematics learning has progressed from elementary to secondary school. They also stressed that it is concerning when students cannot solve fundamental non-routine problems as it may impede their cognitive preparation for the rigorous prerequisites of tertiary-level education.

DISCUSSION

High school leavers bring a plethora of "learned" mathematics content knowledge when they transition to college. However, the findings reveal a significant cognitive gap in the mathematical thinking abilities of these high school leavers entering college. The identified difficulties highlight whether these students are sufficiently prepared for the intellectual demands of college-level mathematics. These difficulties are reflected in a low mean score of 9.15 (SD = 3.84) on the MTT and high rates of incorrect responses to non-routine problems. The results align with earlier research (Nasir et al., 2021), which surprisingly suggests no correlation between students' mathematical grades on national exams and their mathematical thinking ability. The surprising results underscore the need for specific educational interventions, like curriculum changes and targeted problem-solving techniques, to bridge this gap and enhance the alignment of students' mathematical thinking abilities with college level requirements.

The results show how poorly the students could solve mathematical thinking problems due to their inability to use the previously learned school mathematics knowledge. This finding suggests that mathematics is taught in schools as rigid, procedurally-oriented subjects and this was align with previous preconceive notion by researchers (Nasir et al., 2021; Shawan et al., 2021). These consequences could impede meaningful mathematical learning in higher learning, a decrease in classroom participation, completing fewer mathematics credits, and a significant reluctance to enroll in advanced mathematics courses that are essential for the nation's economic development (Ashcraft et al., 2007; Mädamürk et al., 2021). Given the importance of mathematics in developing students' mathematical thinking, these issues are critical in the context of STEM education.

The results of this study suggest that the new KSSM curriculum (Kementerian Pendidikan Malaysia, 2016) has not fully achieved its intended aim of developing a balanced set of knowledge and abilities, including problem-solving, critical thinking, and creative thinking, for the holistic development of students. The introduction section of this paper poses the question, "Is the new curriculum effectively equipping students for college readiness through the provision of a challenging mathematics curriculum?" and receives a negative response. The result of this study implies that although a comprehensive education system may have been implemented since the launch of the Malaysian Education Blueprint (2013-2025) curricular, its actual implementation is inadequate, especially when it comes to preparing students for the rigorous intellectual demands of college-level mathematics. These results underscore the need for a reevaluation and potential curriculum reform to bridge the cognitive gap identified in students' mathematical thinking abilities (Ashcraft et al., 2007; Moses & Cobb, 2001).

To solve these issues related to the results of the study, action is necessary, especially in terms of developing students' ability to think mathematically. First and foremost, a fundamental change in how mathematics is taught is required, focusing on how "teaching students to think" and "doing mathematics" go hand in hand. This change is significant in light of the educational system's historical fallacy that emphasizes rote learning. Students should be encouraged to acquire concepts, improve procedural abilities, and develop a more profound knowledge of mathematical concepts by providing demanding challenges that foster intellectual growth. This method inadvertently fosters meaningful interaction with mathematical topics and promotes a more thorough learning experience, going beyond just procedural understanding.

Second, we propose a fundamental paradigm in mathematical education, highlighting the real-world applications of heuristic-based problem-solving methods. By aligning "doing mathematics" with the more general objective of "teaching students to think," it challenges the old school system's predominant reliance on rote memorization. Non-routine problems can act as intellectual growth accelerators because they present cognitive demands and challenges, as Polya (2004) suggested. Students' comprehension of mathematical ideas is expanded, and their procedural abilities are strengthened when such tasks are included in the curriculum. This approach fosters meaningful engagement and adds to a more thorough and successful learning experience by going beyond the simple acquisition of procedural knowledge (Devlin, 2013; Liu & Niess, 2006; Treffinger et al., 2008).

Applying heuristics is viewed as a valuable tool offering general strategies to enhance learners' understanding and progress toward problem solutions. Heuristics may seem devoid of intrinsic worth in mathematical contexts but can be highly potent (Polya, 1973). Employing various heuristics, such as searching for patterns, building lists, working backwards, and guessing and checking, promotes active learning, enabling students to grasp concepts and improve procedural skills meaningfully. As advocated by scholars (Devlin, 2013; Liu & Niess, 2006; Treffinger et al., 2008), using heuristics as problem-solving tools for non-routine tasks enhances students' development of mathematical thinking. However, further research is required to examine the effective implementation of these heuristics in fostering students' growth in mathematical thinking.

Drawing on Singapore's success as a potential model based on TIMSS and PISA results over the decade, Clark (2009) outlines several reasons that contribute to the effectiveness of Singapore's mathematics education system. These insights align with our ongoing discussion on problem-solving and heuristics, shedding light on crucial aspects that can enhance students' mathematical thinking. He outlines the following reasons for Singapore's success:

> (1) Problem-solving is embedded in Singapore texts, not as a separate activity but as central to every skill and concept discussion. (2) The problems that Singapore students work on are much more complex than those in standard American texts. Two- and three-step problems are the norm. (3) Non-routine and routine problems are included in every grade level (p. 2).

He further elucidated that Singapore's curriculum heavily emphasizes non-routine problems beyond computation specification. Learners will often need to use several different heuristics to solve these problems. In other words, Clark's (2009) insights into Singapore's successful mathematics education system highlight the central role of problem-solving in the curriculum, incorporating more complex problems and including non-routine and routine problems at every grade level. This approach aligns with established educational research emphasizing problem-solving integration into mathematics education (Polya, 1973; Treffinger et al., 2008). Additionally, exposing students to complex problems and various problem types is recognized in the literature as beneficial for cognitive development (Polya, 1973; Treffinger et al., 2008). Singapore's curriculum stands out for its emphasis on problem-solving and its strategic implementation of diverse and challenging problems, contributing to its success in mathematics education.

Despite the identified cognitive gap in students' mathematical thinking abilities, it is essential to understand the subtle relationship between problem-solving ability and academic achievement gaps. The ability to solve complex problems, mainly through heuristics and diverse problem types, plays a pivotal role in students' academic success (Polya, 1973; Treffinger et al., 2008). While problem-solving alone may not entirely explain academic achievement gaps, it serves as a crucial factor. Furthermore, efforts to develop a positive attitude, improve classroom learning materials, and, most importantly, teachers' problem-solving preparedness are crucial for encouraging all students to feel better about mathematics learning. This teacher's preparedness is vital for successful and meaningful curriculum implementation. The final level of curriculum development involves teachers as the primary implementers. More effort needs to be undertaken by the education ministry to actualize the philosophy of the new curriculum.

CONCLUSION

In conclusion, this study has revealed a considerable cognitive difference in the mathematical thinking ability of high school leavers entering college despite receiving A grades on the national examination. However, the grades does not truly reflect a thorough mastery of previously learned mathematical subject knowledge. These students' poor mean scores on the MTT and significant percentages of incorrect answers to non-routine problems indicate they are unprepared for the rigorous intellectual requirements of college-level mathematics. These ramifications include impeding meaningful mathematical learning, encouraging avoidance behaviors, and decreasing participation in STEM education. The findings underscore the imperative for educational interventions to address this cognitive gap and align students' mathematical thinking skills with the demands of tertiary education. Addressing these challenges becomes pivotal to preparing students better and ensuring their success in the increasingly demanding landscape of college-level mathematics.

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