MODESTUM EURASIA Journal of Mathematics, Science and Technology Education, 2025, 21(3), em2599
ISSN:1305-8223 (online)
OPEN ACCESS Research Paper https://doi.org/10.29333/ejmste/16047

Mentoring and online lesson plan sharing platform contribute to mathematics teachers' growth: Teachers' perspective

Ruti Segal [1*]

[1] Oranim Academic College of Education, Tivon, ISRAEL

Received 15 December 2024 ▪ Accepted 07 February 2025

Abstract

This three-year study aimed to examine how an intervention program that included mentoring alongside documenting and sharing lesson plans (LPs) over a collaborative platform environment contributed to the development of the pedagogical content knowledge of in-service mathematics teachers undergoing their first experience teaching advanced high school mathematics. In the program, 20 experienced teachers (mentors) in 20 high schools accompanied 24 inexperienced teachers (mentees) in the planning, teaching, sharing, and documenting of their advanced mathematics lessons on a specially created, collaborative online platform called RAMZOR and in professional development workshops. The RAMZOR platform enables documenting and sharing LPs and resources. The results indicate that, according to the mentees, the combination of mentoring and collaborative platform led to the development of the professional knowledge required for teaching advanced-level high school mathematics and an increase in their self-confidence, sense of efficacy, and ability to meet curriculum goals.

Keywords: high school mathematics teachers, mentoring, professional knowledge for teaching mathematics, lesson plans, collaboration

INTRODUCTION

Teachers are the key to the success of any educational system (Lee, 2014). Alongside their busy day-to-day work, they are also required to remain updated in the mathematical content presented in often vague and overloaded curriculum schedules (Cai et al., 2020); consistently upgrade their mathematical knowledge, teaching approach and practices; and create and implement lesson plans (LPs) and teaching resources to improve their students' learning opportunities (Treacy et al., 2020). To accomplish this, teachers must be given appropriate autonomy and, more specifically, opportunities to collaborate with their colleagues (Lindqvist et al., 2020; OECD, 2011; Vangrieken & Kyndt, 2020), especially by sharing knowledge through joint lesson planning (Cai et al., 2020; Owen, 2015) or by participating in communities where experienced and novice teachers collaborate to promote professional capital (Cai et al., 2020; Li & Lu, 2024). Such a route for sharing best practices can be a system that enhances teachers' professional development by enabling cooperation, support, and discussion; sharing ideas; exchanging thoughts about teaching; working with colleagues; and receiving emotional support from peers (Desimone, 2009; Li & Lu, 2024; Lindqvist et al., 2020; Owen, 2015). Including a system of mentoring will also enhance such a program (Glazer & Hannafin, 2006; Hudson, 2013).

The background to this research was the sharp decline in the number of students sitting for matriculation at the highest level (5 units) between the years 2006-2014 in Israel. This was the result, among other things, of a severe shortage of teachers willing to teach mathematics at this high level, pointing to a need to develop a professional and independent community of practice (CoP) for mathematics teachers who teach at the advanced level (Oberman, 2018) and where experienced teachers could mentor and support those less experienced.

Based on this quest, mathematics educators in the Technion in Israel developed an R&D project to support secondary school mathematics teachers who were teaching 5-unit-level mathematics for the first time.

✉ Ruti.segal@oranim.ac.il **(*Correspondence)**

Contribution to the literature

- It provides empirical evidence for how a combine intervention using both personalized mentoring and digital collaborative platform can effectively support experienced teachers transitioning to teaching advanced mathematics.
- It extends existing research on teacher mentoring by demonstrating how intensive, long-term mentoring integrated with technological tools can simultaneously develop teachers' subject matter knowledge (SMK), pedagogical content knowledge (PCK), and emotional well-being.
- It offers practical insights into how online collaborative lesson sharing digital platforms can be leveraged to create sustainable communities of practice among mathematics teachers, that preserve and accumulate mathematical knowledge for teaching (MKT), while fostering professional growth.

Support was initially implemented via mentoring, collaboratively documenting and sharing LPs, and professional development workshops. The mentor and mentee taught in the same school: the mentee had years of experience teaching lower-level mathematics and the mentor was well-experienced in teaching at a higher level. The collaborative aspect involved planning detailed LPs utilizing a digital environment platform developed specifically for this purpose ('RAMZOR', details below).

The current study focuses on the contribution that mentoring using the RAMZOR platform makes to the mentees' professional knowledge.

THEORETICAL BACKGROUND

Mentoring as a Lever for Teacher Collaboration

Mentoring can involve interaction between teachers and teacher educators, between peers, and between novice and experienced teachers who can be providers of

> the profession, modelers of practice, supporters of reflection, gleaners of knowledge … (Clarke et al., 2014, p. 163).

The research literature provides extensive discussion of the advantages of the mentoring model in schools to enable support, professional development, collaboration, and implementation of educational programs and initiatives (e.g., Clarke et al., 2014; Kadji-Beltran et al., 2014; Li & Lu, 2024; Lu et al., 2020; Mullen & Klimaitis, 2021). Mentoring involves collaboration between a mentor and mentee that includes reflection, exposure to one another's' teaching methods, practicing autonomous decision making, and mutual professional development and practice (Lu et al., 2020; Michailidi & Stavrou, 2021; Richter et al., 2013; Shanks et al., 2022). Mentoring can involve novice and expert teachers, pre- or in-service teachers and teacher educators, or peers. It can foster powerful teaching, increasing skills via teamwork, and the exchange of information and proficiencies (Glazer & Hannafin, 2006; Hudson, 2013).

Mentors need to listen with empathy, share their experience, and encourage their mentees to develop insight through reflection (Clutterbuck, 1991; Kennedy, 2005). In the classic setup, the mentor will be an experienced teacher who addresses the mentee's needs throughout the initial stages of teaching (Handrianto et al., 2022; Lim et al., 2018; Michailidi & Stavrou, 2021). Mentoring can also be expanded to larger circles of collaboration by building communities of mentors and mentees, thus inviting increased interaction with other teachers and more exposure to innovative teaching materials. An added benefit of extended circles of collaboration is combatting isolation and the creation of professional learning communities (Clarke, et al., 2014; Kadji-Beltran et al., 2014; Li & Lu, 2024; Shanks et al., 2022).

Mentoring can increase the mentee's content and pedagogic awareness for planning and teaching a subject (Segal et al., 2023); curricular knowledge; their repertoire of teaching skills; efficacy; their confidence to deliver relevant, quality teaching; and their self-confidence in dealing with the challenges of teaching (Kadji-Beltran et al., 2014; Lim et al., 2018; Vangrieken et al., 2017). Even experienced veteran teachers can benefit from mentoring: it can assist them in focusing on new aspects of classroom interactions and they may learn innovative pedagogical approaches from their mentee's recent academic studies (Segal et al., 2023; Smith, 1996). Effective mentoring entails two-way constructive feedback rather than one-directional dictation and mutual respect of each other as colleagues (Glenn, 2006; Kadji-Beltran et al., 2014). Both partners develop professionally (Vangrieken & Kyndt, 2020). Thus, mentoring is beneficial for all teachers, not only novice, as a manifestation of lifelong learning where teachers face mathematical and pedagogical challenges (Handrianto et al., 2022; Li & Lu, 2024).

The Challenge of Creating Lesson Plans

It may be said that designing LPs is at the heart of teachers' professional work. To design effective LPs, teachers require the complex mathematical and pedagogical knowledge that spans the subjects and levels taught (Ball et al., 2008; Cai et al., 2020; Davis &

Simmt, 2006) and, most importantly, be able to integrate this knowledge into the lessons (Cai et al., 2020). The challenge begins early in the teacher-training phase and continues during their daily in-service work (Cai et al., 2020; Cevikbas et al., 2023; Lim, 2018). It requires thoughtful consideration and discretion (Boote, 2006; Movshovitz-Hadar, 2018). However, many teachers consider preparing detailed written LPs to be an 'unnecessary burden' during their pre-service education and tend to abandon the practice 'in the real world', preparing their LPs spontaneously. As a result, practical knowledge is seldom shared with colleagues at the community level (Movshovitz-Hadar et al., 2014), and the teaching profession suffers from 'collective amnesia' (Shulman, 1987, p. 11, in Cai et al., 2020) despite the widely recognized benefits of sharing knowledge through joint lesson planning (Lim et al., 2018; Owen, 2015).

The challenges to preparing effective LPs are considerable. Curriculum knowledge is one obstacle, given the frequent changes in curricula (Brown, 2009; Cai et al., 2020) and the fact that curricula are often written in vague, ambiguous terms. Insufficient pedagogical and mathematical knowledge can be another barrier to constructing meaningful lessons and making appropriate discretionary decisions (Boote, 2006; Shriki & Lavy, 2012). Another challenge, as Lim et al. (2018) and Huang et al. (2016) have pointed out, is whether teachers are familiar with their students' mathematical skills.

These challenges can be managed through collaboration in planning and teaching LPs, which can improve novice teachers' efficacy over time (Bauml, 2014) and develop their mathematical and pedagogical knowledge (Sullivan, 2018). Writing an LP in collaboration with peers involves shared thinking, brainstorming, and exposure to novel pedagogical ideas. This may arouse the teacher's curiosity and expand their pedagogical repertoire for designing appropriate learning tasks and adequate learning materials, leading to more interesting learning opportunities for their students and enhancing their confidence as teachers (Owen, 2015; Ruys et al., 2012).

Professional Knowledge for Teaching Mathematics

The skills and knowledge teachers require include pedagogical knowledge (teaching materials and methods), didactic knowledge, the ability to analyze the impact of their actions (Lim, 2018; Shriki & Lavy, 2012), and most importantly, the ability to integrate them into their preparation of LPs and other teaching materials.

In 1987, Shulman (1987), in an attempt to closely define the types of knowledge teachers require, introduced the terms 'PCK' and 'SMK'.

Later, Ball and Bass (2003, 2009) and Ball et al. (2008) defined the term 'MKT', which includes various sub-components of SMK and PCK. For example, SMK includes 'specialized content knowledge' (knowledge and skills unique to teaching mathematics, such as the thoughtful integration of examples, awareness of various way to solve a problem, explanatory skills, evaluation procedures, how to identify and represent mathematical concepts, generalizations of patterns, and more) and 'horizon content knowledge' (awareness of how mathematical topics are related throughout the curriculum and the entire mathematical landscape, which may not be part of the curriculum but is useful to learning). PCK includes 'knowledge of content and curriculum' (how mathematics content is organized in the curriculum, which topics need to be taught at each stage, etc.), and 'knowledge of content and teaching' (how to design a lesson to successfully impart the mathematical knowledge, for instance, by choosing simpler examples to start a lesson), 'knowledge of content and students' (how students think, what may confuse them, what may be easier or more difficult for them, how to spark student interest, how to lead students to the correct solution and connect concepts, how to interpret student response, etc.).

Mathematics teachers' PCK is continuously developing, a result of their teaching actions alongside belonging to and collaborating with a community of mathematics teachers. Carlson et al. (2019) presented the 'refined consensus model' in which they divided PCK into three distinct yet dynamically interrelated realms: collective PCK (the amalgam of knowledge formed by combining the teacher's personal knowledge and teaching experiences with the documented knowledge and experience of other professionals); personal PCK (the sum of the individual teacher's PCK reflected by the teacher's previous teaching and learning experiences and possibly including knowledge personally garnered from colleagues, scholars or other content specialists during professional exchanges or professional development), and enacted PCK (the knowledge and skills that the teacher has specifically sought out to use in the student setting and for a goal).

A teacher with excellent MKT will offer learners mathematical richness beyond what is offered in the classroom textbook (Herbst & Kosko, 2014; Hill et al, 2008; Ma, 1999; Santagata & Lee, 2021; Wilkie, 2014).

Study Aims

Based on the above information, we developed a program and follow-up study that implemented mentoring with a special collaborative platform (called RAMZOR, see below) for documenting and sharing LPs for supporting in-service mathematics teachers during their first experience in teaching high school mathematics at the 5-unit level. The program focused on the challenge of preparing appropriate LPs and documenting, implementing, and sharing them on the RAMZOR platform. The follow-up study aimed to

assess the mentees evaluation of the program based on how they felt it had enhanced their knowledge and skills in preparing LPs and teaching accordingly.

THE INTERVENTION PROGRAM

Precursors to the Development of the Program

Between 2008 and 2014, Israel saw a sharp decline in the number of students taking 5-unit-level mathematics matriculation exams (Blass, 2020; Davidovitch & Yavich, 2018). The main reason for this was a severe shortage of teachers willing to teach 5-unit mathematics (Blass, 2022). Teacher educators realized that, to alleviate this situation, mathematics teachers need to be provided with opportunities to develop the required skills for teaching high-level high school mathematics. Alongside a specialized program, they recognized a need to encourage the development of a platform that would allow collaboration between teachers (in effect, a professional CoP for mathematics teachers).

Matriculation Levels

There are three levels of mathematics matriculation exams in Israel. All levels include algebra, differential and integral calculus, analytical geometry, verbal problems, series, probability, and statistics. The higher levels (four and five) teach them more in-depth and at a higher level of complexity. Levels four and five also include plane geometry using deductive proofs, differential and integral calculus of quotient functions, trigonometric functions, and exponential functions, and level five also adds subjects such as vectors and complex numbers (Oberman, 2018). The questions on the matriculation exams at level five require solid understanding of the subjects alongside the connectivity between different concepts, such as the connection between vectors and spatial trigonometry, between complex numbers and series, and more.

To gain admission to in-demand faculties at universities, students must pass the five-unit-level mathematics matriculation exam (Ben-David & Kimhi, 2020; Fried et al., 2018; Zeedan & Hogan, 2022). Therefore, many students are eager to succeed at this level, and the State of Israel is interested in ensuring that all students have access to learning opportunities at this level.

The Intervention Program

In order to increase the number of teachers qualified to teach the 5-unit level, a program was initiated whereby teachers already experienced in teaching level-3 and -4 mathematics and were interested in teaching level 5 were assigned mentors to supply the support needed for this step. Following three preliminary design studies (Movshovitz-Hadar et al., 2014; Movshovitz-Hadar, 2018; Segal et al., 2016, 2017, 2020; Shriki & Movshovitz-Hadar, 2011), a three-year (2014-2016) intervention program called 'RAMZOR in the north' project was designed to encourage mathematics teachers to embrace teaching 5-unit mathematics, thereby increasing the number of qualified teachers.

The project comprised three cornerstones:

(1) organized mentoring to upgrade teachers' knowledge and confidence in teaching high-level math,

(2) 'RAMZOR', a collaborative digital environment that provided mathematics teachers with a platform to prepare, document, and share teaching materials, and

(3) professional development workshops ('RAMZOR' is the digital platform; 'RAMZOR in the north' is the name of the three-pronged project, one aspect of which is use of the RAMZOR platform.)

Mentors and mentees

The project pairs teachers who are teaching 5-unit-level mathematics for the first time (mentees) with teachers who already have experience (at least five years) teaching this level. The mentees, although experienced mathematics teachers for lower levels (at least five years), now have to face teaching a larger number of topics at the highest level of complexity and preparing their students for the important matriculation exams. The mentors assist the mentees in preparing teaching programs (periodic or thematic) and designing LPs, teaching resources, and evaluation items. Mentees and mentors also observed each other's lessons. Periodic face-to-face or online meetings took place.

The RAMZOR digital environment

The teachers (both mentors and mentees) were introduced to the RAMZOR platform which had been designed in 2014. The 'structured' capabilities of the country-wide platform include advanced search engines, monitoring capabilities, and permissions management. It allows collaborative development of teaching and learning materials to encourage professional interaction among teachers.

Information is entered into pre-determined Word-file templates designed for LPs, thematic or periodic programs, assessment items, or evaluation tests, each of which is preceded by several descriptive details (name of lesson, prior knowledge required, grade and study level, learning objectives, anticipated difficulties and how to manage them, and more). The LP template itself includes a detailed reference to each part of the lesson, including time allotted, flow (form of teacher-students dialogue), and more. Auxiliary materials such as worksheets, homework assignments, and links to relevant applets can be attached. The RAMZOR

environment also allows user comments, feedback, and elaboration for each entry. Each teacher can choose with whom to share materials, ranging 'for their eyes only' or the entire community. This environment allows teachers' knowledge toward specific class grades and levels and specific learning goals to be preserved, shared, and accumulated while implementing specific learning opportunities (as suggested by Cai et al., 2020). More details about RAMZOR can be found in Movshovitz-Hadar's (2018) book on mathematical education in Israel.

The mentors and mentees were asked to use the RAMZOR platform for composing LPs, planning teaching units (by topic or time), and developing assessment items/exams for the various topics in the curriculum. Thus, teachers could examine, adapt, and implement LPs designed by their peers (the mentees in the other schools) and for their own classes. In the first, second, and third years, the mentees were focused on teaching grades 10, 11, and 12, respectively, and were asked to write 10, 8, and 5 LPs each year, respectively. The mentors also shared their lessons. The mentors and mentees could work independently or collaboratively and choose with whom they wanted to share their LPs at every stage of their work.

Professional development workshops

A team of expert mathematics educators from the Technion Institution (including the author of this paper) led the workshops. During each of the three years of the project, the mentees and mentors (sometimes together, sometimes as dedicated groups) participated in approximately 8-10 online or face-to-face workshops/meetings with the project staff (the author was a member of this staff).

The mentees' workshops covered the 5-unit-level mathematics curriculum (with which they were not yet familiar): structure, possible challenges, different teaching approaches to consider. The mentors' workshops included updates in the structure of the curriculum, challenges involved in guiding mathematics teachers at the 5-unit level, various approaches to mentoring teachers, and considerations and challenges involved in designing LPs and teaching materials.

As part of the joint workshops, the teachers learned how to integrate technological tools and environments into teaching mathematics; various ways to solve problems; enrichment and extension topics from experts, such as the link between music and mathematics; how to read articles and translate them into LPs; how to analyze student solutions to questions from matriculation exams; and how to re-design 4-unit level problems in the textbooks and matriculation exams to make them applicable for the 5-unit level.

METHODOLOGY

Research Questions

The overall study attached to the project involved many aspects. In this paper, we focus on the following two questions:

1. What did the mentees perceive to be the most valuable contribution(s) of the 'RAMZOR to the north' project to their professional knowledge for teaching 5-unit-level mathematics?
2. What did the mentees perceive to be the most valuable contribution(s) of collaboratively writing, documenting, and sharing LPs via the RAMZOR platform to their professional knowledge for teaching 5-unit-level mathematics?

Participants

Participants were recruited through a voluntary program in which schools were invited to nominate experienced expert teachers who might be willing to act as mentors and to suggest teachers who were not experienced in teaching 5-unit level mathematics who might be willing to participate as mentees. The study thus involved 20 mentors and 24 mentees from 20 high schools in the northern district of Israel (in four instances, one mentor was assigned to two teachers in the school).

All the participants were informed that this was a three-year project and expressed their commitment to complete it. Final selection of participants was made by the administration of the Northern District, which also included the supervisor of mathematics teaching in the Northern District on behalf of the Ministry of Education.

In the first year of the project, the mentees taught 5-unit-level mathematics in grade 10. They continued with these same students in the following years (grade 11 and grade 12). The mentor-mentee pair continued for the duration of the study. The mentors, who were also teaching grade 11 and grade 12 throughout the duration of the project, used the RAMZOR platform to prepare these lessons, and they served as the infrastructure for the mentees for the next two years. **Table 1** presents demographic information.

Data Collection

A mixed method (Creswell, 2014) protocol was used to gather comprehensive data from the mentees. The following three tools were used.

Formative assessment questionnaires

This questionnaire comprised Likert-type statements and open questions. It was administered periodically throughout the study (10 in the first year, 8 in the second, and 4 in the third) to provide ongoing insight into the participants' perceptions of the project. Questions

Table 1. Demographic data of participants (percentages are approximate)

Attribute	Mentees	Mentors
Years of seniority: M (SD)	13 (8 years)	22 (12 years)
Origin of bachelor's degree (BA, BSc, or BEd)	48%–Department of mathematics (university)	80%–Department of mathematics (university)
	27%–Department of mathematics education (college or university)	10%–Department of mathematics education
	13%–Engineering	10%–Engineering
	13%–Other studies in mathematics: statistics, economics, business administration, etc.	
Origin of master's degree (MA, MSc, or MEd)	9%–Mathematics (university)	25%–Mathematics (university)
	39%–Mathematics education (college or university)	35%–Mathematics education (colleges or university)
	9%–Other (education, women's studies, etc.)	40%–Other (education, statistics, counselling, etc.)

Note. M: Mean; SD: Standard deviation; & 43% do not hold a master's degree

related to the RAMZOR platform for implementing and sharing LPs, workshop content, weekly mentor-mentee meetings, observations of others' lessons, and feelings regarding teaching high school mathematics at this level.

Two examples of the Likert-type statements are "To what extent did your participation in the workshop contribute to your mathematical/pedagogical knowledge?" and "To what extent did the weekly meeting with your mentor contribute to your mathematical/pedagogical knowledge?"

Some examples of open-ended questions: Do you plan to integrate the content/lecture/activity presented in the workshop in your teaching in the future? Have you written LPs on the RAMZOR website in the past month? If so, how many and on what topics? Did you share your LPs on RAMZOR website? If so, how many and on what topics? Have you observed any of your mentor's lessons in the past month? What were the lesson topics and what insights did you gain? Did you find LPs on the topics you were looking for on the RAMZOR website?

Summary questionnaire

Distributed at the culmination of the three-year period, this comprehensive questionnaire included 13 Likert-type statements (scale 1-5). In the first question, the mentees were asked to rate the degree of contribution that each of the six components of the RAMZOR to the north project–mentoring, the RAMZOR website, the workshops, the teaching schedule, consultations with the project team, and consultations with other mentors and mentees–made. Next came six statements related to mentoring and six statements to assess the contribution that various aspects of collaborative LP-writing using the RAMZOR platform made to the mentees' professional development (see results below for exact statements). Also included were 10 open-ended questions in which the respondents could freely elaborate on their responses in the first part.

Some examples of the open-ended questions: Describe how mentoring has improved your ability to teach mathematics at the 5-unit level? How has mentoring contributed to your mathematical knowledge? How has mentoring contributed to your pedagogical knowledge? How has collaboratively writing and sharing LPs with other mentors and mentees on the RAMZOR website contributed to your mathematical knowledge/pedagogical knowledge/professional development as a teacher of mathematics at the 5-unit level?

Semi-structured interviews

At the end of the three-year project, seven mentees, chosen at random, were interviewed individually (either face-to-face or remotely). Each interview lasted 60-90 minutes. It allowed the mentees to verbally express what they felt were the most valuable contributions of the three elements of the project to their professional development and thus allowed the researcher to better understand, refine, and develop various aspects. Interviews were audio-recorded and transcribed for analysis.

Example of questions asked during the interview to the mentees:

1. Describe your feelings about your participation in RAMZOR to the north project as a teacher who is teaching 5-unit-level math for the first time.
2. How have the collaborative LPs helped you succeed in teaching 5-unit-level mathematics?
3. What contribution has mutual lesson observations made to your professional development regarding teaching mathematics at the 5-unit level?
4. What contribution has writing and sharing LPs on the RAMZOR website made to your professional development?

5. What contribution have the workshops made during the project made to your professional development?

Data Analysis

The Likert-type statements in both the formative assessment and summary questionnaires were analyzed quantitatively for frequencies, percentages, means, and standard deviations.

The open-ended questions and the one-on-one interviews were analyzed using systematic content analysis: the responses were divided into 'units of meaning' (one idea represented by anything from one word to a sentence or paragraph) and themes were identified and categorized using the 'constant comparative analysis' method (Charmaz, 2014; Glaser & Strauss, 2017). These were divided into primary and secondary content areas (Fram, 2013). The qualitative data analysis helped better understand and refine the insights and conclusions of the quantitative results, and vice versa.

The combination of periodic formative assessment questionnaires and summary questionnaires enabled a comprehensive longitudinal analysis of the mentees' professional development over the three-year period. While the summary questionnaires provided an overall evaluation of the project's impact, the systematic collection of formative assessment data throughout the project via repeated questionnaires (10 in the first year, 8 in the second, and 4 in the third) allowed us to track subtle changes in the mentees' perceptions, confidence levels, and professional growth at different stages of their journey. This dual analytical approach – combining end-of-project summative data with ongoing formative assessments – provided rich insights into both the final outcomes and the developmental progression of the mentees' experiences with mentoring and the RAMZOR platform.

The results of the analyses were validated and verified by five judges (experts in mathematics education) until 80% agreement was reached regarding the categorization.

RESULTS

The results focus on the mentees' perspective. For each research question, the summary questionnaire analysis is presented first and then the analysis of the formative assessment questionnaires.

Question 1. What did mentees perceive to be the most valuable contribution(s) of the 'RAMZOR to the north' project to their professional knowledge for teaching 5-unit-level mathematics?

Summary Questionnaires

The results of the summary questionnaire show that three components–mentoring, the RAMZOR website, and the workshops–received the highest ranking (see **Figure 1**).

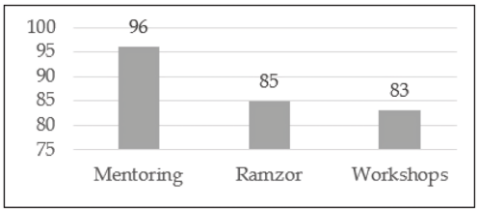

Figure 1. The three components of the RAMZOR project (of six) that received the highest scores in the summary questionnaire showing the percentage of mentees' who rated them 'agree' or 'strongly agree' (Source: Author's own elaboration)

It can be clearly seen that the mentees perceived mentoring to have the greatest contribution to their knowledge for teaching mathematics at the 5-unit level.

A closer analysis of the summary questionnaires and interviews reinforce this. **Table 2** presents the average scores (and standard deviations) of the participants' responses to the six statements in the summary questionnaire regarding the contribution of mentoring to the mentees' professional development.

The open questions and interviews indicated that the weekly mentor-mentee meetings seemed especially valuable. The mentees commented that these meeting improved their ability to prepare LPs, teaching resources, and assessment items and provided a professional and emotional anchor during which they could ask for help with any difficulties. It allowed them to become more familiar with the 5-unit level curriculum (documented on RAMZOR), improve their knowledge

Table 2. Contribution of mentoring to various aspects of professional knowledge according to mentees (question 1-question 6 of summary questionnaire)*

No	Statement: Mentoring helped … for teaching mathematics at the 5-unit level	Mentees: M (SD)
1	Strengthen my sense of competence.	4.39 (0.89)
2	Improve my ability to prepare tests and assessment items.	4.21 (0.90)
3	Improve my capability to meet the teaching schedule of the 5-unit curriculum.	4.17 (0.83)
4	Improve my ability to prepare LPs.	4.04 (1.10)
5	Improve my ability to prepare teaching.	4.04 (1.10)
6	Improve my mathematical understanding of the various topics.	4.00 (1.00)

Note. M: Mean; SD: Standard deviation; & *Scored from 1 (strongly disagree) to 5 (strongly agree)

about subjects they had not yet taught on this level, become adept at how to convert the curriculum into an LP that considered difficulties they may encounter, and feel safe to navigate this complex process and deal with any frustrations. Some answers support this:

> I was teaching vectors for the first time, a new and complex subject for me. Without the mentor's assistance I would have had difficulty understanding the subject and creating appropriate LPs.

> Our weekly meetings allowed me to deepen my knowledge about the curriculum. Also, I discovered interesting connections between the subjects while collaborating via RAMZOR. For example, in my LP, I presented a solution to a spatial geometry task and my mentor showed me a different solution method using vectors.

Formative Assessment Questionnaires

The formative assessment questionnaires tracked the mentees' perspective throughout the project and complements the findings noted above by showing how the mentees' attitudes regarding the value of the mentoring changed over the three years with respect to

(1) complying with the 5-unit level teaching schedule,
(2) managing teaching at this level,
(3) filling gaps in their mathematical and pedagogical knowledge, and
(4) feeling emotionally supported and more self-confident.

Compliance with the 5-unit level teaching schedule

Most first-year mentees constantly consulted the mentor and depended on the mentor to review their LPs (via RAMZOR). In the second year, one mentee noted,

> I started writing LPs via RAMZOR more independently.

However, they still depended on their mentor's comments 'to help improve the plan if necessary'. In the third year, they felt more independent, but still needed the mentor's support for complex topics.

Managing teaching

In the first year, the mentees depended heavily on the mentor. For example, mentees noted that their mentors helped them know 'what to pay attention to next time', 'what exercises should be added', 'what students might have difficulty with', and 'how to lead the students to learn from the errors they make'.

In the second year they continued to need extensive assistance but were starting to feel the burgeoning feelings of independence and even enough confidence to try methods that may not have matched the mentor's approach.

In the third year, 93% of the mentees reported greater independence in managing their teaching. One mentee noted that her sense of independence had

> improved since last year. I now know what material to convey and what to emphasize.

Filling gaps in mathematical and pedagogical knowledge

In the first year, over half of the mentees admitted to gaps in the required mathematical knowledge and noted that the mentors help them fill in these gaps. By the second year, they felt more confident in their own ability to fill in the gaps in knowledge and would turn to other sources such as the RAMZOR website or other sites. They also reported that the workshops helped them upgrade their didactic knowledge, such as using GeoGebra to help their students understand complex subjects.

In the third year, the mentees exhibited more confidence in their knowledge and their ability to fill in any gap. However, they also noted the increased complexity of the subject matter, requiring consultation with the mentor.

Emotional support from the mentors

This was specifically noted and valued, especially in the first half of the first year:

> … someone who can help me at any time.

> I speak to my mentor every day for at least a few minutes.

> … it gives me a sense of security, I'm not alone.

In the second year, 87% of the mentees reported that they felt more confidence. However, they still felt the pressure and responsibility of having to prepare students for matriculation and were happy to have the support of their mentor. In the third year, 87% also reported increased self-confidence but still depended on the emotional support the mentors provided:

> Without the mentoring, coping would have been more difficult.

> For every dilemma, I had a partner who helped and supported me.

The source of the mentees' increasing independence probably lies in the teaching experience they acquired and their experience of success after submitting their students to the first part of the matriculation exams at the end of the second year (11th grade) of the project:

Table 3. Value of collaboration in writing LPs on the RAMZOR according to the mentees (average scores for question 7-question 12 of summary questionnaire)*

No	Statement	Mentees: M (SD)
7	Writing LPs collaboratively with *my mentor* on the RAMZOR website contributed to my professional development for teaching 5-unit-level mathematics for the first time.	4.54 (0.83)
8	My 3-year experience writing LPs on the 'RAMZOR' website with other mentors and mentees *expanded my mathematical knowledge*.	4.25 (0.89)
9	My 3-year experience writing LPs on the 'RAMZOR' website with other mentors and mentees *expanded my pedagogical knowledge*.	4.25 (0.84)
10	Preparing a detailed lesson plan is necessary for teaching in the classroom.	4.13 (1.30)
11	My 3-year experience writing LPs on the 'RAMZOR' website has strengthened/empowered me as a teacher.	4.04 (1.04)
12	Writing LPs collaboratively with *other mentees* on the RAMZOR website contributed to my professional development for teaching 5-unit-level mathematics for the first time.	3.79 (1.25)

Note. M: Mean; SD: Standard deviation; & *Scored from 1 (strongly disagree) to 5 (strongly agree)

> After we received the grades of the exam, I realized that I indeed had the ability to teach five units. This improved my self-confidence significantly.

Their new-found independence was also due to overt or covert messages from the mentors:

> I feel that my mentor trusts me 100 percent compared to last year. Already in the second year, my mentor said that he now trusts me as a teacher for 5 units and I don't need to consult him as often.

Despite that, by the end of their second year, even though many mentees felt confident in their progress, teaching the 12th grade (third year of the project) was still challenging as it included different and more complex subjects compared to previous years and therefore a continued relationship with the mentor was still as essential as ever.

One notable observation was the mentees' growing sense of independence. In the first year, all but one reported that they fully depended on the mentor; in the second year, two-thirds (16/24) reported that they felt more independent compared to the previous year; in the third year, almost all (21) reported that they felt more independent compared to the previous year.

Question 2. What did mentees perceive to be the most valuable contribution(s) of collaboratively writing, documenting, and sharing LPs via the RAMZOR platform to their professional knowledge for teaching 5-unit-level mathematics?

Summary Questionnaires

Six statements in the summary questionnaire pertained to the contribution that collaborative LP writing on the RAMZOR website made to the mentees' ability to teach 5-unit-level mathematics. **Table 3** presents the average scores and standard deviations for these statements.

As can be seen from **Table 3**, the mentees felt that collaborating with the mentor was the most important aspect in their professional development, and collaborative writing with other mentees was the least. An indicative quote:

> My mentor was very familiar with my class … During our collaboration via RAMZOR, he suggested how to adapt my lessons to my students. This was very effective.

However, they also valued the support they received on the website from other mentors and mentees (statement 8 and statement 9). For example,

> While preparing LPs in RAMZOR, another mentor pointed out some connections between different subjects in the curriculum that I was unaware of. For example, she showed me how to solve Euclidean geometry problem using trigonometry, which can sometimes lead to a simpler solution.

> Another mentor helped me pose an exam question appropriate for this level: combining function parameters and adding questions that link the value of the parameter to graph.

> I didn't properly understand the connection between a function and its derivatives. A mentor (not mine) in a workshop noticed my difficulty and explained, step by step, how it can be presented in an LP and how to teach it.

As noted earlier (see section, the challenge of creating LPs), the preparation of detailed LPs was not always valued as important by many teachers. However, 65% of the participants scored statement 10 (the importance of detailed LPs) '4' or '5' and 22% '3', indicating that a good majority now understood the importance of a detailed LP for providing an optimal lesson. Some statements indicating their change in attitude:

> I learned to focus on clearly formulated lessons.

> Writing detailed lesson plans help me see how they fit within the sequence of topics and organize my teaching according to the schedule.
>
> Writing LPs forced me to devote time to thinking what difficulties my students may have. It allowed me to organize and optimize my lesson.

With respect to empowerment (statement 11), sample quotes include:

> Using a prepared template (in RAMZOR) helps me make more precise use of mathematical terms and accurately manage my time.
>
> I felt very professional documenting my LPs in RAMZOR.
>
> I found it challenging, [but] it allowed me to feel more prepared. I entered the class with more self-confidence.

Formative Assessment Questionnaires

Analysis revealed the mentees' increased appreciation of collaborative LP writing, documentation, and sharing via the RAMZOR platform. They acknowledged that it allowed them to recognize the advantages of writing an LP before the lesson and summarizing it after, promoted collaboration with colleagues (mentors and mentees), and exposed them to new mathematical and pedagogical ideas as a result of reviewing the LPs of others.

To elaborate in more detail, at the beginning of their first year on the project, the mentees did not write detailed LPs using RAMZOR, as they had not been in the habit of doing so when teaching the lower levels. However, the challenge of teaching 5-unit-level mathematics made them realize the need to carefully plan and document their LPs. At first, many mentees reported having difficulty writing a detailed LP on the RAMZOR website and complained about the amount of time required. However, as the first year progressed, there were more and more statements in the vein of

> I really enjoy writing LPs in RAMZOR. It's my creation.
>
> This is an important and wonderful process creating and sharing a quality product adapted to my class.
>
> Success depends on advance preparation.
>
> When you plan a lesson using the RAMZOR format, you can return to the LP after the lesson and make changes if needed.

Writing their LP also contributed to the discovery of new insights:

> My mentor's comments and previous experience led me to emphasize certain points.

During the second year, despite the extensive experience the mentees had gained in writing LPs on the RAMZOR website, about half reported that it was not easier for them compared to the previous year. The interviews indicated that this was due to the added complexity of the mathematical subjects, the mentees' newfound awareness of the complex thought processes required, and because they had become more aware of their collaborative responsibility of 'depositing' lessons on the website:

> I was aware that my colleagues had access to them, so writing them demanded more time and attention.

On the other hand, the other half reported that it was easier for them this year due to the experience they had gained and the feedback they had received last year from the project team and their peers.

In the final year, about 71% of the mentees observed that preparing a detailed LP increases the quality of teaching and efficient use of class time:

> The goal can be better achieved.
>
> I save time in the lesson and present interesting materials beyond what is suggested in textbook and curriculum.
>
> It improves my degree of readiness and the lesson's quality.

In fact, as time passed, the mentees noted that sharing on RAMZOR accelerated their path to writing high-quality, accurate LPs and encouraged them to write original material, thus expanding their pedagogical knowledge:

> Other teachers will be viewing them, so I try to make my lessons as clear and detailed as possible.
>
> Knowing I am sharing encourages me to make them special. I look for ideas on the web, and this exposes me to new ideas.

They were also aware that the RAMZOR website exposed them to new, enriching mathematical and pedagogical ideas and that

> the LPs on RAMZOR are an opportunity for each of us to learn from the experience of others.
>
> You see several ways of teaching the same subject, and can choose what we think is most effective.
>
> You don't have to reinvent the wheel every time. It's great!

Table 4. Range of mentees' scores: How reviewing and using other teachers' LPs contributed to their mathematical and pedagogical knowledge (percentages, N = 24)

	Very large extent	Large extent	Medium extent	A little extent	Not at all
Mathematical knowledge	22	64	14	0	0
Didactic knowledge	22	71	7	0	0

The following quote from one of the mentees towards the end of the third year not only demonstrates the development process but how the process led to greater enthusiasm.

> During my participation in the project, I went through a very significant process. I used to teach mathematics according to the textbook. I did not turn to other sources. Thanks to my mentor and thanks to my work on the RAMZOR website, I was exposed to mathematical content that I had not encountered and novel teaching approaches.
>
> In the first year, I checked with my mentor at every stage of my lesson preparation. Then I started looking for ideas on RAMZOR, and this gave me ideas for planning my lessons. For example, I recently taught about infinite geometric series. In the textbook, this topic is presented using 'dry' formulas and rules. I knew my class would have a hard time with it because the whole idea is rather counter-intuitive. However, I found an LP on RAMZOR that helped me [better understand the didactics] and then another with a story about an infinite geometric series. I combined them into an 'improved' LP of my own and uploaded it to the website. I also found and added a video from an online site. All this really piqued my curiosity about the topic, which led me to read more articles and become interested in fractals.

In the formative assessment questionnaires, the mentees were also asked to score how reviewing and making use of other teachers' LPs (on RAMZOR) contributed to their mathematical and didactic knowledge'. **Table 4** presents the distribution of their scores at the end of the third year of the project.

As can be seen in **Table 4**, all the mentees believed that being exposed to other teachers' LPs contributed to their mathematical/didactic knowledge at least to a medium extent. It should be remembered that for the mentees, this was their first experience in teaching high-level mathematics, so this result is not surprising. The following are several illustrative quotations from the mentees regarding specific mathematical content:

> I found LPs that exposed me to new ways of proving a theorem involving geometry locus and I learned how homothetic transformation enables the preservation of the loci features.
>
> One LP gave a clear visual explanation for the meaning of the integral formula. It opened my eyes.
>
> I applied an LP on similar triangles using the problem-posing approach. The students' difficulties helped me understand how students understand the topic and process information.
>
> I tried an LP that showed the solution of distance word problems graphically. I realized it helped many students understand the verbal formulation and identify the connections between the given.

CoP

Although not specifically discussed in this paper, in the overall study we also assessed the contribution that belonging to a CoP made. At the end of the third year, the responses to the statement 'sharing my LPs with other teachers contributed to my sense of belonging to the teacher CoP' were, as follows: 29% of participants believed this to a very large extent, 43% to a large extent, and 28% to a medium extent. None considered it ineffective. Some representative quotations:

> Sharing LPs on RAMZOR gets me constructive feedback from colleagues.
>
> This is a good opportunity to share our ideas and products and get. This is the meaning of a community.
>
> When I share my LPs on RAMZOR I feel that I am contributing to other teachers.
>
> Sharing my LPs empowers me as a teacher.

DISCUSSION AND CONCLUSIONS

This study investigated the contribution of the three-year 'RAMZOR to the north' mentoring project to the professional development of mathematics teachers who were teaching advanced (5-unit level) high school mathematics for their first time. In general, the findings indicated the importance of one-on-one mentoring and the RAMZOR online platform.

Contribution of Mentoring

The results of the current study emphasized that the mentees in the current study '[stand] on the shoulders of those who have gone before them' (Cai et al., 2020, p. 130). In other words, the help of experienced mentors

was crucial in developing the professional knowledge of the mentees who were just starting out at the higher level. A mentoring partnership between experienced and inexperienced teachers who teach in the same school and are teaching the same subject at the same level is a meaningful factor for supporting novice teachers' professional growth and emotional well-being, as also found by Kwok et al. (2021) and which aligns with previous research highlighting the benefits of mentoring between teachers in the same school (Kadji-Beltran et al., 2014; Shanks et al. 2022).

Our study showed that the mentoring process contributed to the teachers' SMK and PCK (as conceptualized by Ball et al., 2008 in their MKT framework). All the mentees reported significant improvements in their understanding of complex mathematical topics and their ability to structure LPs according to the curriculum. This development in SMK is crucial for teaching 5-unit-level mathematics, as teachers must be familiar with major disciplinary ideas and structures (Zazkis & Mamolo, 2011). The mentoring process also enhanced mentees' PCK, such as knowledge content and teaching, by improving mentees' ability to design and implement LPs for the loaded 5-unit mathematics curriculum.

The findings of the current research also extend the results of Lu et al. (2020) and Richter et al. (2013) by demonstrating how intensive, ongoing, and supportive mentoring can increase a teacher's professional competence and well-being by developing the specialized knowledge required for teaching advanced level mathematics. In the present study the mentoring included utilizing the online RAMZOR collaborative platform that allows the unique blend of autonomous and collaborative interaction in teacher development. This addressed Vangrieken and Kyndt (2020) findings on the importance of balancing autonomy and collaboration in teacher development. The RAMZOR platform allowed mentees individual autonomy when writing and documenting their LPs and collaborative interaction when sharing and reviewing others' LPs.

Contribution of Collaborative Writing, Documenting, and Sharing LPs Over the RAMZOR Platform

The RAMZOR platform emerged as a key factor in enhancing mentees' professional knowledge, aligning with research emphasizing the importance of collaboration in teacher professional development, since it allows exposure to others pedagogical and mathematical resources ideas (Desimone, 2009; Li & Lu, 2024; Owen, 2015). Our findings suggest that digital platforms such as RAMZOR can serve as effective tools for knowledge sharing and community building among mathematics teachers and expose teachers to original, innovative, and useful teaching resources from others, thus creating professional capital (Li & Lu, 2024) of advanced level mathematics teachers.

Furthermore, the process of co-creating, documenting, teaching, and sharing LPs on the RAMZOR website contributed to the development of what Carlson et al. (2019) term 'collective PCK' for teaching advanced level mathematics. Such a collective knowledge base, in our case accessible through the RAMZOR platform, allows teachers to gain new perspectives on teaching approaches and strategies, which in turn develops into their 'personal PCK'. In addition, teaching the LPs further developed the teachers' 'enacted PCK', as they had to learn to deal with time constraints, student questions, and difficulties in real classroom settings.

This study's results also expand the work of Cai et al. (2020) by demonstrating how a digital platform can enable the preservation, accumulation, and sharing of teaching knowledge, thereby allowing novice practitioners to build on the expertise of their more experienced colleagues. This process of knowledge sharing and collaborative development aligns with Wenger's (1998) and Moodley's (2019) concepts of the CoP, suggesting that platforms such as RAMZOR can support the development of a mathematics teachers' CoP which, according to Wenger's (1998) definition, requires participants to

(1) have a shared domain of interest (in our study, teaching advanced level mathematics),

(2) engage in joint activities and discussions to help one another, share information, and build relationships (utilizing mentoring), and

(3) develop a shared repertoire of resources and experiences (utilizing the RAMZOR platform).

Implications for Teacher Professional Development

The findings of this study have several important implications for practical projects dedicated to teachers' professional development, especially in the context of supporting those who are teaching specific, unfamiliar content for the first time.

1. A combination of one-on-one mentoring and digital collaboration can be highly effective in supporting teachers' professional growth. Therefore, teacher education programs and school districts should consider implementing a similar structure.

2. The potential of digital tools (such as RAMZOR) for facilitating teacher collaboration and knowledge sharing should be exploited, meaning that educational institutions should consider adopting or developing similar platforms to support teachers ongoing professional development. Such a platform can also reduce gaps between schools since teachers from advanced schools can contribute high-quality

pedagogical and mathematical ideas to teachers from less advanced schools (Li & Lu, 2024).

3. The value of collaborative lesson planning as a form of professional development should be underscored; teacher education programs should consider incorporating collaborative lesson planning activities, supported by digital platforms, into their curricula.

4. Fostering CoPs can be valuable for teacher professional development. Schools and districts should consider how to affect CoPs to support the development of subject-specific teacher communities, both within and across schools.

Limitations and Future Research

This study provides valuable insights into the combined contribution of mentoring and a collaborative platform for writing, documenting, and sharing LPs to mentees' professional development. However, it has several limitations that should be addressed in future research.

1. This study focused on the specific context of the Israeli education system and a specific level of mathematics. This may limit the direct applicability of the methods studied to other educational contexts.

2. Futures studies might explore how mentoring and the RAMZOR platform (or similar) can alleviate the challenges teacher of other subjects or grades face during their daily workload and compare the results with ours.

3. Participation in this study of the 'RAMZOR to the north' project was voluntary, meaning that the participants may have been highly motivated teachers, and the results cannot be generalized for all mathematics teachers.

4. Our data was based solely on the participants' evaluation of the project. Also, we did not examine if and how this translated into changes in classroom practice or student outcomes. Future research could explore these connections more explicitly.

5. Finally, it would be interesting if future research would examine the contribution of such a project to the mentor's professional development.

Despite these limitations, the current study contributes valuable insight into the potential of mentoring combined with collaborative lesson planning over technological platforms for enhancing mathematics teachers' professional development.

Funding: No funding source is reported for this study.

Ethical statement: The author stated that the study was conducted in accordance with institutional ethics guidelines while maintaining participants anonymity. The study was approved by the institutional ethics committee of Oranim Academic College of Education in January 2024 (Approval code: 210). Written informed consent was obtained from participants.

Declaration of interest: No conflict of interest is declared by the author.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the author.

REFERENCES

Ball, D. L, & Bass, H. (2009). *With an eye on the mathematical horizon: Knowing mathematics for teaching to learners' mathematical futures* [Paper presentation]. The 43rd Jahrestagung der Gesellschaft fur Didaktik der Mathematic.

Ball, D. L., & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. In B. Davis, & E. Simmt (Eds.), *Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group* (pp. 3-14). CMESG/GCEDM.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*, 389-407. https://doi.org/10.1177/0022487108324554

Bauml, M. (2014). Collaborative lesson planning as professional development for beginning primary teachers. *The New Educator, 10*(3), 182-200. https://doi.org/10.1080/1547688X.2014.925741

Ben-David, D., & Kimhi, A. (2020). Education quality and future wages: The impact of math matriculation quality in the periphery and center of Israel. *SSRN*. https://doi.org/10.2139/ssrn.3764939

Blass, N. (2020). Achievements and gaps: The status of the Israeli education system. *Taub Center*. https://www.taubcenter.org.il/wp-content/uploads/2020/12/achievementsangapseng.pdf

Blass, N. (2022). Achievements in Israel's education system: An overview. State of the nation report: Society, economy and policy. *Taub Center*. https://www.taubcenter.org.il/wp-content/uploads/2022/12/Education-Overview-ENG-2022.pdf

Boote, D. N. (2006). Teachers' professional discretion and the curricula. *Teachers and Teaching: Theory and Practice, 12*(4), 461-478. https://doi.org/10.1080/13450600600644319

Brown, M. W. (2009). The teacher-tool relationship: Theorizing the design and use of curriculum materials. In J. T. Remillard, B. A. Herbel-Eisenman, & G. M. Lloyd (Eds.), *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 17-36). Routledge.

Cai, J., Morris, A., Hohensee, C., Hwang, S., Robison, V., Cirillo, M., Kramer, S. L., Hiebert, J., & Bakker, A. (2020). Addressing the problem of always starting over: Identifying, valuing, and sharing professional knowledge for teaching. *Journal for Research in*

Mathematics Education, 51(2), 130-139. https://doi.org/10.5951/jresematheduc-2020-0015

Carlson, J., Daehler, K. R., Alonzo, A. C., Barendsen, E., Berry, A., Borowski, A., Carpendale, J., Kam Ho Chan, K., Cooper, R., Friedrichsen, P., Gess-Newsome, J., Henze-Rietveld, I, Hume, A, Kirschner, S., Liepertz, S., Loughran, J., Mavhunga, E., Neumann, K., Nilsson, P., ..., & Wilson, C. D. (2019). The refined consensus model of pedagogical content knowledge in science education. In A. Hume, R. Cooper, & A. Borowski (Eds.), *Repositioning pedagogical content knowledge in teachers' knowledge for teaching science* (pp. 77-94). Springer. https://doi.org/10.1007/978-981-13-5898-2_2

Cevikbas, M., König, J., & Rothland, M. (2023). Empirical research on teacher competence in mathematics lesson planning: Recent developments. *ZDM–Mathematics Education, 56*, 101-113. https://doi.org/10.1007/s11858-023-01487-2

Charmaz, K. (2014). *Constructing grounded theory*. SAGE.

Clarke, A., Triggs, V., & Nielsen, W. (2014). Cooperating teacher participation in teacher education: A review of the literature. *Review of Educational Research, 84*(2), 163-202. https://doi.org/10.3102/0034654313499618

Clutterbuck, D. (1991) *Everyone needs a mentor: Fostering talent at work*. Institute of Personnel Management.

Creswell, J. W. (2014). *Qualitative, quantitative and mixed methods approach*. SAGE.

Davidovitch, N., & Yavich, R. (2018). The effect of motivation and self-efficacy on math studies in the Israeli Ministry of Education's program "give five". *International Journal of Higher Education, 7*(6), 63-77. https://doi.org/10.5430/ijhe.v7n6p63

Davis, B., & Simmt, E. (2006). Mathematics-for-teaching: An ongoing investigation of the mathematics that teachers (need to) know. *Educational Studies in Mathematics, 61*, 293-319. https://doi.org/10.1007/s10649-006-2372-4

Desimone, L. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher, 38*(3), 181-199. https://doi.org/10.3102/0013189X08331140

Fram, S. M. (2013). The constant comparative analysis method outside of grounded theory. *Qualitative Report, 18*, Article 1.

Fried, M. N., Perl, H., & Arcavi, A. (2018). Highlights in the development of education and mathematics education in the state of Israel: A timeline. In N. Movshovitz-Hadar (Ed.), *Israel mathematics education K-12* (pp. 3-19). World Scientific Publishing. https://doi.org/10.1142/9789813231191_0001

Glaser, B. G., & Strauss, A. L. (2017). *The discovery of grounded theory: Strategies for qualitative research*. Routledge. https://doi.org/10.4324/9780203793206

Glazer, E. M., & Hannafin, M. J. (2006). The collaborative apprenticeship model: Situated professional development within school settings. *Teaching and Teacher Education, 22*(2), 179-193.

Glenn, W. J. (2006). Model versus mentor: Defining the necessary qualities of the effective cooperating teacher. *Teacher Education Quarterly, 33*(1), 85-95.

Handrianto, C., Jusoh, A. J., Syuraini, S., Rouzi, K. S., & Alghazo, A. (2022). The implementation of a mentoring strategy for teachers' professional development in elementary school. *Elementary: Islamic Teacher Journal, 10*(1), 65-80. https://doi.org/10.21043/elementary.v10i1.13676

Herbst, P., & Kosko, K. (2014). Mathematical knowledge for teaching and its specificity to high school geometry instruction. In J.-J. Lo, K. R. Leatham, & L. R. Van Zoest (Eds.), *Research trends in mathematics teacher education* (pp. 23-45). Springer. https://doi.org/10.1007/978-3-319-02562-9_2

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26*(4), 430-511. https://doi.org/10.1080/07370000802177235

Huang, R., Gong, Z., & Han, X. (2016). Implementing mathematics teaching that promotes students' understanding through theory-driven lesson study, *ZDM, 48*(3), 425-439. https://doi.org/10.1007/s11858-015-0743-y

Hudson, P. (2013). Mentoring as professional development: 'Growth for both' mentor and mentee. *Professional Development in Education, 39*(5), 771-783. https://doi.org/10.1080/19415257.2012.749415

Kadji-Beltran, C., Zachariou, A., Liarakou, G., & Flogaitis, E. (2014). Mentoring as a strategy for empowering education for sustainable development in school. *Professional Development in Education, 40*(5), 717-739. https://doi.org/10.1080/19415257.2013.835276

Kennedy, A. (2005). Models of continuing professional development: A framework for analysis. *Journal of In-service Education, 31*(2), 235-250. https://doi.org/10.1080/13674580500200277

Kwok, A., Rios, A. R., Keese, J., Suárez, M., Worley, C., Patterson, M., Mitchell, D., & Huston, D. (2021). Match games: Exploring the match between novice teachers and induction coaches. *Teachers and Teaching, 27*(1-4), 246-268. https://doi.org/10.1080/13540602.2021.1939003

Lee, W. O. (2014). Comparative analysis of high performing education systems: Teachers, teaching, and teacher education as factors of success. In S. K. Lee, W. O. Lee, & E. L. Low (Eds.), *Educational policy innovations: Levelling up and sustaining educational achievement* (pp. 217-229). https://doi.org/10.1007/978-981-4560-08-5_13

Li, X., & Lu, L. (2024). School quality matters: How mentoring can be restructured to support beginning teachers in different schools from a social capital perspective. *Research Papers in Education*. https://doi.org/10.1080/02671522.2024.2368496

Lim, W., Son, J. W., & Kim, D. J. (2018). Understanding preservice teacher skills to construct lesson plans. *International Journal of Science and Mathematics Education, 16*(3), 519-538. https://doi.org/10.1007/s10763-016-9783-1

Lindqvist, H., Weurlander, M., Wernerson, A., & Thornberg, R. (2020). Conflicts viewed through the micro-political lens: Beginning teachers' coping strategies for emotionally challenging situations. *Research Papers in Education, 35*(6), 746-765. https://doi.org/10.1080/02671522.2019.1633559

Lu, X., Kaiser, G., & Leung, F. K. S. (2020). Mentoring early career mathematics teachers from the mentees' perspective–A case study from China. *International Journal of Science and Mathematics Education, 18*(7), 1355-1374. https://doi.org/10.1007/s10763-019-10027-y

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teacher's understanding of fundamental mathematics in China and the United States*. Lawrence Erlbaum Associates. https://doi.org/10.4324/9781410602589

Michailidi, E., & Stavrou, D. (2021). Mentoring in-service teachers on implementing innovative teaching modules. *Teaching and Teacher Education, 105*, Article 103414. https://doi.org/10.1016/j.tate.2021.103414

Moodley, M. (2019). WhatsApp: Creating a virtual teacher community for supporting and monitoring after a professional development programme. *South African Journal of Education, 39*(2), 1-10. https://doi.org/10.15700/saje.v39n2a1323

Movshovitz-Hadar, N. (2018). Mathematics teachers documenting, sharing, and improving their work using a newly developed software. In N. Movshovitz-Hadar (Ed.), *K-12 mathematics education in Israel–Issues and innovations* (pp. 311-316). World Scientific Publishing. https://doi.org/10.1142/9789813231191_0034

Movshovitz-Hadar, N., Shriki, A., & Zohar, O. (2014). *Collaborative structuring of the pedagogical content knowledge accumulating within mathematics teachers' community of practice* [Paper presentation]. The 2nd Jerusalem Conference on Research in Mathematics Education.

Mullen, C. A., & Klimaitis, C. C. (2021). Defining mentoring: A literature review of issues, types, and applications. *Annals of the New York Academy of Sciences, 1483*(1), 19-35. https://doi.org/10.1111/nyas.14176

Oberman, J. (2018). Israeli mathematics curricula K-12 (Section 4: High school grades 10-12). In N. Movshovitz-Hadar (Ed.). *K-12 mathematics education in Israel–Issues and innovations* (pp. 424-429). World Scientific Publishing.

OECD. (2011). *Building a high-quality teaching profession: Lessons from around the world*. OECD Publishing. https://doi.org/10.1787/9789264113046-en

Owen, S. M. (2015). Teacher professional learning communities in innovative contexts: 'Ah hah moments', 'passion' and 'making a difference' for student learning. *Professional Development in Education, 41*(1), 57-74. https://doi.org/10.1080/19415257.2013.869504

Richter, D., Kunter, M., Lüdtke, O., Klusmann, U., Anders, Y., & Baumert, J. (2013). How different mentoring approaches affect beginning teachers' development in the first years of practice. *Teaching and Teacher Education, 36*, 166-177. https://doi.org/10.1016/j.tate.2013.07.012

Ruys, I., van Keer, H., & Aelterman, A. (2012). Examining pre-service teacher competence in lesson planning pertaining to collaborative learning. *Journal of Curriculum Studies, 44*(3), 349-379. https://doi.org/10.1080/00220272.2012.675355

Santagata, R., & Lee, J. (2021). Mathematical knowledge for teaching and the mathematical quality of instruction: A study of novice elementary school teachers. *Journal of Mathematics Teacher Education, 24*(1), 33-60. https://doi.org/10.1007/s10857-019-09447-y

Segal, R., Lehavi, Y., & Merzel, A. (2023). Improving the professional awareness of mathematics teachers and teacher instructors using video-based curiosity-driven discourse – A case study. *International Journal of Science and Mathematics Education*, 1-24. https://doi.org/10.1007/s10763-023-10418-2

Segal, R., Shriki, A., & Movshovitz-Hadar, N. (2016). Facilitating mathematics teachers' sharing of LPs. In C. Csikos, A. Rausch, & J. Szitanyi (Eds.), *Proceedings of the 40th Conference of PME – The International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 171-178). Szeged, Hungary.

Segal, R., Shriki, A., & Movshovitz-Hadar, N. (2017) Mentoring and collaborating on lesson plans as a means for training mathematics teachers towards

teaching higher levels [Oral communication]. In B. Kaur, W. K. Ho, T. L. Toh, & B. H. Choy (Eds.), *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 267). PME.

Segal, R., Shriki, A., & Movshovitz-Hadar, N. (2020). 'RAMZOR – A digital environment that constitutes opportunities for mathematics teachers collaboration. In H. Borko & D. Potari (Eds.), *The Proceedings of the 25th ICMI Conference: Teachers of Mathematics Working and Learning in Collaborative Groups* (pp. 692–699). ICMI.

Shanks, R., Attard Tonna, M., Krøjgaard, F., Annette Paaske, K., Robson, D., & Bjerkholt, E. (2022). A comparative study of mentoring for new teachers. *Professional Development in Education, 48*(5), 751-765. https://doi.org/10.1080/19415257.2020.1744684

Shriki, A., & Lavy, I. (2012). Perceptions of Israeli mathematics teachers regarding their professional development needs. *Professional Development in Education, 38*(3), 411-433. https://doi.org/10.1080/19415257.2011.626062

Shriki, A., & Movshovitz-Hadar, N. (2011). Nurturing a community of practice through a collaborative design of lesson plans on a Wiki system. *Interdisciplinary Journal of e-Skills and Lifelong Learning, 7*, 339-357. https://doi.org/10.28945/1528

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22. https://doi.org/10.17763/haer.57.1.j463w79r56455411

Smith, J. P. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education, 27*(4), 387-402. https://doi.org/10.2307/749874

Sullivan, P. (2018) Supporting teachers in improving their knowledge of mathematics. *The Journal of Mathematical Behavior, 51*, 161-166. https://doi.org/10.1016/j.jmathb.2017.08.006

Treacy, P., Prendergast, M., & O'Meara, N. (2020). A 'new normal': Teachers' experiences of the day-to-day impact of incentivising the study of advanced mathematics. *Research in Mathematics Education, 22*(3), 233-248. https://doi.org/10.1080/14794802.2019.1668832

Vangrieken, K., & Kyndt, E. (2020). The teacher as an island? A mixed method study on the relationship between autonomy and collaboration. *European Journal of Psychology of Education, 35*(1), 177-204. https://doi.org/10.1007/s10212-019-00420-0

Vangrieken, K., Meredith, C., Packer, T., & Kyndt, E. (2017). Teacher communities as a context for professional development: A systematic review. *Teaching and Teacher Education, 61*, 47-59. https://doi.org/10.1016/j.tate.2016.10.001

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge University Press. https://doi.org//10.1017/CBO9780511803932

Wilkie, K. J. (2014). Upper primary school teachers' mathematical knowledge for teaching functional thinking in algebra. *Journal of Mathematics Teacher Education, 17*, 397-428. https://doi.org/10.1007/s10857-013-9251-6

Zazkis, R., & Mamolo, A. (2011). Reconceptualizing knowledge at the mathematical horizon. *For the Learning of Mathematics, 31*(2), Article 813.

Zeedan, R., & Hogan, R. E. (2022). The correlation between budgets and matriculation exams: The case of Jewish and Arab schools in Israel. *Education Science, 12*(8), Article 545. https://doi.org/10.3390/educsci12080545