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Exploratory teaching: Integrating applet to teach arithmetic multiplication operation

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Abstract
Foundational arithmetic skills in early education and understanding effective teaching methodologies are fundamental. This paper studies how exploratory teaching (ET) impacts the integration of an applet and its effect on understanding multiplication in a second-grade primary school. The data collected assessed students' knowledge levels and overall performance. The intervention sessions involved tasks in an applet to understand the meanings of the multiplication operation. The ET model was structured with five sessions, each with four phases. The results indicated that this method significantly improved students' arithmetic self-efficacy and effectively integrated applets into the classroom for enhanced learning of arithmetic operations.

Keywords: digital technology, applets, exploratory teaching, arithmetic operations, primary school

INTRODUCTION

Mathematics has been referred to as a challenging discipline to comprehend (Isoda & Olfos, 2021). Multiplication teaching is common in many countries' curricula (Isoda & Olfos, 2021). In Portugal, the teaching of arithmetic operations, specifically multiplication, begins in the 2nd grade of primary school. It is acknowledged in academic literature that students commonly face challenges throughout the learning journey (Isoda & Olfos, 2021; Ribeiro & Almeida, 2022; Ribeiro et al., 2023). Active utilization of digital artifacts by students can facilitate learning (Lopes & Costa, 2019). This approach encourages greater student engagement and contributes to better performance outcomes (Jahnke et al., 2022). According to Canavarro et al. (2012), current curriculum demands require students to have the opportunity to learn in environments oriented toward meaningful tasks, allowing them to make meanings of mathematical knowledge. Aligned with this idea, exploratory teaching (ET) emerges as a form of interactive pedagogical intervention centered on the active learning of the student (Canavarro et al., 2012; Oliveira et al., 2013). This study therefore aims to answer the following research question: How does the Multiplication applet on the Hypatiamat platform promote understanding of the meanings of the arithmetic operation multiplication among students in the 2nd year of primary school? This study follows the ET approach and addresses the need to understand how to best integrate digital artifacts into the classroom. The study was conducted in a second-grade class to address the following research question: What is the influence of ET on the inclusion of an applet in the classroom and on the understanding of the concepts of the arithmetic operation of multiplication in second-grade students at primary school? To address the research question, we accomplished the following objectives:

(1) identify students' difficulties regarding the meanings of the arithmetic operation of multiplication,

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Contribution to the literature

- This study introduces an innovative intervention approach that effectively incorporates digital artifacts into classroom settings, resulting in notable improvements in learning outcomes.
- It showcases a pedagogical strategy grounded in formative assessment principles, employing tasks and rubrics to continuously evaluate and enhance student performance. It addresses a significant educational challenge concerning students' struggles with multiplication, as evidenced by prior research and observed among primary school students.
- The research examines the impact of exploratory intervention on students' comprehension of multiplication's fundamental concepts, shedding light on its educational efficacy.

(2) analyze how ET influences the integration of the Multiplication applet in the classroom, and

(3) examine the influence of this inclusion on understanding the meanings of the arithmetic operation multiplication.

This article provides insights into how the integration of digital artifacts through an exploratory approach can benefit the learning of the meanings of the arithmetic operation of multiplication.

THEORETICAL FOUNDATION

Arithmetic Operation of Multiplication

Learning multiplication should be based on understanding its meanings (Mendes, 2012). According to Ribeiro and Almeida (2022), multiplication has three meanings: successive addition of equal installments, rectangular configuration and combinatorics. Successive addition of equal parts is the first sense to be explored (Maffia & Mariotti, 2018) and should be the main way of introducing arithmetic operation multiplication (Gómez, 1991). Successive addition of equal parts is associated with repeated addition. For example, $2 + 2 + 2 + 2 = 4 \times 2 = 8$ (Ribeiro et al., 2023). If we consider the operation $4 \times 2 = 8$, the first quantity (4) corresponds to the number of times the second quantity (2) will be repeated, which remains invariable (Ribeiro & Almeida, 2022). It should be noted here that the representations 4×2 and 2×4 have different meanings, even though they have the same product (8) (Ribeiro & Almeida, 2022) Rectangular configuration involves elements that can be arranged in a rectangular format to determine their quantity (Ribeiro & Almeida, 2022; Ribeiro et al., 2023). Thus, this direction involves a certain number of rows, each row with the same number of objects (Troutman & Lichtenberg, 1995). The aim is to find the total number of objects. However, it is important to realize that changing the order of the factors will imply a change in meaning, even if the product is the same. For example, the representations 4×2 and 2×4 represent 4 lines with 2 objects or 2 lines with 4 objects (Ribeiro & Almeida, 2022). In the third sense of multiplication (combinatorial), elements from different sets are combined to calculate the total number of possible combinations (Ribeiro et al., 2023; Troutman & Lichtenberg, 1995). Let's consider the operations 4×2 and 2×4 and adapt them to the example given by Troutman and Litchtenberg (1995). We can conclude that they mean, respectively, combining 4 types of sandwiches with 2 types of drinks, or 2 types of sandwiches with 4 types of drinks. Ribeiro and Almeida (2022) consider that the two representations represent the "same thing and are ways of combining all the elements of one set with all the elements of another disjoint set" (p. 64). Several authors state that children usually have difficulties learning multiplication (Kouba & Franklin, 1995; Ribeiro & Almeida, 2022). Gómez (1991) states that problems involving successive addition are easier for students. However, when solving problems involving the successive addition of equal installments, students find it difficult to distinguish the meaning of, for example, 4×2 and 2×4 in a given context. It is therefore essential that students have the opportunity to solve different problem situations to understand that the same product can be obtained from different contexts (Ribeiro & Almeida, 2022). In the rectangular configuration, difficulties arise when using the important information inherent in the images that accompany the statements (Ribeiro & Almeida, 2022) and the situations associated with the numbers used in the contexts of the tasks (Ribeiro & Almeida, 2022). In the combinatorial sense, it is common for students to have difficulties understanding how combinations occur and how they can obtain the total of these combinations by calculating the arithmetic operation of multiplication (Ribeiro & Almeida, 2022). Gómez (1991) states that it is in this sense of multiplication that students experience the most difficulties.

Digital Artifacts

Digital artifacts, such as applets, contribute to engagement among students in mathematics class. The interaction between students and digital artifacts can support the understanding of mathematical content (Long & Bouck, 2023; Martins et al., 2019).

Applets typically present mathematical problems and provide rewarding feedback, which can help students understand the problem through visualization (Gorev & Gurevich-Leibman, 2015). The Hypatiamat project in Portugal has a platform, the Hypatiamat platform, which contains a variety of applets and games aimed at learning math from the 1st to the 9th year of school (Pinto et al., 2022). The Multiplication applet is a digital artifact from the Hypatiamat platform based on the math curriculum and organized according to the multiplication theme and the transversal competencies of the curriculum. This applet, aimed at the second and third years of primary school, contains 63 frames that combine problem situations and multiplication-related explanatory frames (Pinto et al., 2022). The applet can provide feedback to students, orientated towards the achievement of their work, through virtual rewards received via a points and medals system (Verdasca et al., 2020). The Multiplication applet is intuitive, allowing students to play an active and autonomous role according to their learning pace. Students can repeat each task as many times as they need to with hints, appropriate feedback and adapted solution proposals, thus leading the student to understand and appropriate the concept (Verdasca et al., 2020).

The mere use of artifacts does not generate learning (Lopes & Costa, 2019; Viberg et al., 2023). To ensure student learning, teachers need to monitor the process (Shin et al., 2017). A study conducted by Viberg et al. (2023) in Sweden examined the integration of digital artifacts in mathematics education, involving 68 students aged between 17 and 18, and three teachers from three schools. Teachers were invited to use the digital artifact MathAid as they deemed most appropriate in their teaching practices. The results revealed a lack of student monitoring during the use of the digital artifact, highlighting the need for guided integration. For the utilization of digital artifacts to benefit students' learning, the teacher needs to select the most suitable artifact (Gorev & Gurevich-Leibman, 2015) and promote structured and meaningful use (Martins, 2020). Among the ways digital artifacts can be utilized, their utility as epistemic tools stands out (Lopes & Costa, 2019). The transition to an epistemic tool occurs when the artifact is used to promote learning and construct knowledge (Lopes & Costa, 2019). The use of digital artifacts as epistemic tools depends on the epistemic mediation of the teacher (Costa et al., 2021). As artifacts become epistemic tools, the quality of learning increases (Silva, 2021). Technology has predominantly been used in the classroom in a passive manner (Office of Educational Technology, 2024). Consequently, students often assume the roles of observers and consumers of information (Jahnke et al., 2022; Office of Educational Technology, 2024). Students must engage in "meaningful mathematical tasks that allow them to reason mathematically about important ideas and make meaning of mathematical knowledge" (Canavarro et al., 2012, p. 256). Active utilization of technology enables students to take ownership of their learning, collaborate with peers, become autonomous, and engage in discussions (Jahnke et al., 2022; Office of Educational Technology, 2024). Classroom discussion plays an important role in mathematics learning (Huang & Sutherland, 2022).

Exploratory Teaching

A lesson can be structured in various ways or comprise multiple moments (Ferreira & Ponte, 2017). Organizing the learning environment following the ET model allows students to engage actively and autonomously in challenging mathematical tasks (Jesus et al., 2020). In an exploratory lesson, there are moments of collective work, during which students work autonomously while the teacher monitors their work and provides guidance when necessary (Oliveira et al., 2013; Jesus et al., 2020). In this teaching approach, the role of the teacher is to monitor and facilitate students' learning (Canavarro et al., 2012), similar to learning environments with technology (Viberg et al., 2023).

An ET lesson typically consists of three or four phases (Canavarro et al., 2012; Ponte & Quaresma, 2020). Canavarro et al. (2012) outline the four phases comprising this teaching model: the "task introduction phase", the "task development phase", the "task discussion phase", and the "mathematical learning systematization phase". Each phase includes the actions and intentions of the teacher, regarding the promotion of mathematical learning and classroom management (Canavarro et al., 2012). In the "task introduction phase," the teacher presents the tasks to the students and the tools that should be used (Canavarro et al., 2012). According to Shimizu et al. (2010), appropriate tasks enable students to deepen their knowledge and stimulate their learning. This initial phase is crucial for the development of students' autonomous work. Students need to have a clear understanding of the context and objectives of the task. To achieve this, the teacher should attentively listen to the comments and questions of the students (Canavarro et al., 2012). In the "task development" phase, students work individually or in groups, assuming an active and autonomous role. The teacher's function is to monitor students' work and provide necessary guidance. This guidance can be given through questions that allow students to reflect on their solutions/errors (Canavarro et al., 2012; Oliveira et al., 2013). In this task development phase, students need to produce materials such as written records, as they become objects of discussion (Canavarro et al., 2012; Martins, 2020). In the "task discussion phase", different solutions proposed by students are confronted, compared, and discussed (Canavarro et al., 2012). This phase is highlighted by Ferreira and Ponte (2017), Guerreiro et al. (2015), Ponte (2017), and Ponte and Quaresma (2020) as an important moment for understanding mathematical content. This moment allows students to communicate, share, analyze, and compare their solutions, reflecting on similarities and differences among various procedures (Ferreira & Ponte, 2017). During these moments of sharing solutions, the

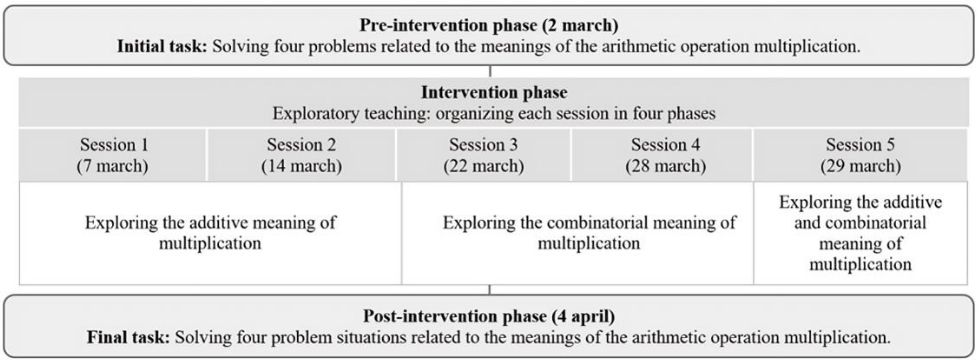
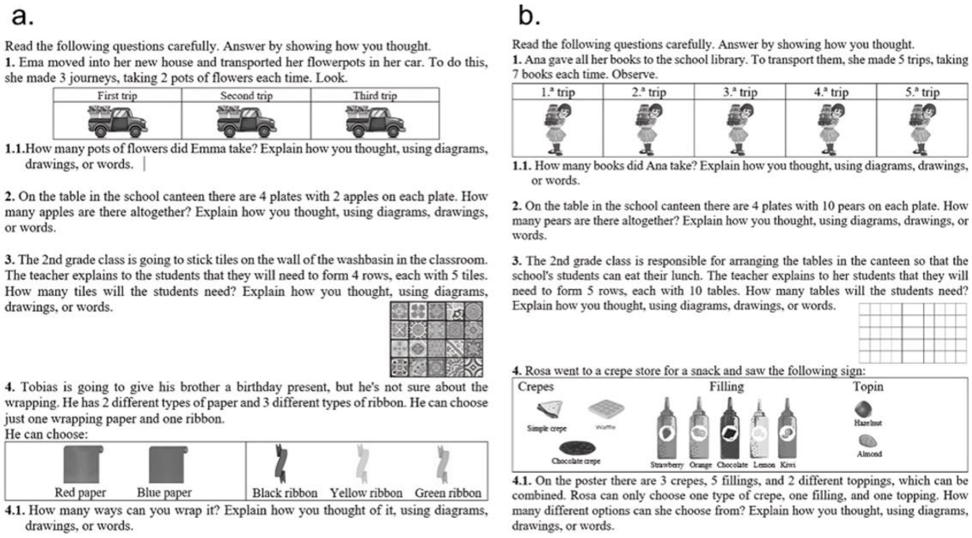


Figure 1. Pre-Intervention, intervention, and post-intervention phases (Source: Authors' own elaboration)



a.

Read the following questions carefully. Answer by showing how you thought.

1. Ema moved into her new house and transported her flowerpots in her car. To do this, she made 3 journeys, taking 2 pots of flowers each time. Look.

First trip	Second trip	Third trip

1.1. How many pots of flowers did Emma take? Explain how you thought, using diagrams, drawings, or words.

2. On the table in the school canteen there are 4 plates with 2 apples on each plate. How many apples are there altogether? Explain how you thought, using diagrams, drawings, or words.

3. The 2nd grade class is going to stick tiles on the wall of the washbasin in the classroom. The teacher explains to the students that they will need to form 4 rows, each with 5 tiles. How many tiles will the students need? Explain how you thought, using diagrams, drawings, or words.

4. Tobias is going to give his brother a birthday present, but he's not sure about the wrapping. He has 2 different types of paper and 3 different types of ribbon. He can choose just one wrapping paper and one ribbon.

He can choose:

Red paper, Blue paper | Black ribbon, Yellow ribbon, Green ribbon

4.1. How many ways can you wrap it? Explain how you thought of it, using diagrams, drawings, or words.

b.

Read the following questions carefully. Answer by showing how you thought.

1. Ana gave all her books to the school library. To transport them, she made 5 trips, taking 7 books each time. Observe.

1.ª trip	2.ª trip	3.ª trip	4.ª trip	5.ª trip

1.1. How many books did Ana take? Explain how you thought, using diagrams, drawings, or words.

2. On the table in the school canteen there are 4 plates with 10 pears on each plate. How many pears are there altogether? Explain how you thought, using diagrams, drawings, or words.

3. The 2nd grade class is responsible for arranging the tables in the canteen so that the school's students can eat their lunch. The teacher explains to her students that they will need to form 5 rows, each with 10 tables. How many tables will the students need? Explain how you thought, using diagrams, drawings, or words.

4. Rosa went to a crepe store for a snack and saw the following sign:

Crepes: Simple crepe, Waffle, Chocolate crepe
Filling: Strawberry, Orange, Chocolate, Lemon, Kiwi
Topin: Hazelnut, Almond

4.1. On the poster there are 3 crepes, 5 fillings, and 2 different toppings, which can be combined. Rosa can only choose one type of crepe, one filling, and one topping. How many different options can she choose from? Explain how you thought, using diagrams, drawings, or words.

Figure 2. (a) Initial task sheet; and (b) final task sheet (Source: Authors' own elaboration)

teacher needs to induce mathematical discussion, as it doesn't occur spontaneously (Guerreiro et al., 2015). In the "mathematical learning systematization" phase, the teacher's discourse is more directive, as a synthesis of the learnings worked on throughout the lesson is presented (Canavarro et al., 2012).

When technology is integrated into the classroom with an appropriate pedagogical structure, mathematics learning becomes meaningful (Sedaghatjou & Rodney, 2018).

MATERIAL AND METHODS

This research is of a mixed nature (Cohen et al., 2017), interpretative (Amado, 2017) and follows an action research design (Bogdan & Biklen, 2013). Data collection and analysis were conducted using techniques and instruments typical of both quantitative and qualitative research (Cohen et al., 2017). The quantitative research findings are expounded upon and complemented by qualitative data through an exhaustive description and interpretation of the data (Amado, 2017).

Context and Participants

This research took place in a 2nd grade class of primary school comprising 24 students (11 female and 13 male, aged between seven and eight) from a primary school in Portugal. Data collection was conducted by a trainee teacher (TT) from a teacher training course at a higher education institution in Portugal. The implementation of the research had the consent of all guardians, students, and educational entities involved.

Data Collection

The data collection took place between March and April 2022, spanning three phases: pre-intervention, intervention, and post-intervention phases, as illustrated in **Figure 1**. All data collected (written records, photographs, and audio recordings) were used exclusively for this study, following the acquisition of informed, freely given, and clarified consent from all guardians of education, and authorization from the school group. Furthermore, the anonymity of all involved participants was maintained, in strict compliance with the Declaration of Helsinki.

In the pre- and post-intervention phases, initial and final tasks were administered, aimed at obtaining feedback on students' knowledge regarding the meanings of the arithmetic operation of multiplication through the assessment of tasks (Andrade, 2000; Brookhart & Chen, 2015) (**Figure 2**).

Table 1. General criteria

Level 1 (L_1)	Level 2 (L_2)	Level 3 (L_3)	Level 4 (L_4)
The solution does not demonstrate an understanding of the mathematical concepts involved or does not provide a response.	The solution demonstrates a limited understanding of the mathematical concepts involved and contains numerous inaccuracies.	The solution shows some Understanding of the mathematical concepts involved and contains some inaccuracies.	The solution demonstrates a thorough understanding of the mathematical concepts involved in the task.

Table 2. Specific descriptions (task 1, 2,3 with additive meaning while task 4 has combinatorial meaning)

Task	LK	Objective 1	Objective 2	Objective 3
1, 2, & 3	L_1	Not presenting a solution or presenting a solution that has nothing to do with the problematic situation.	Not presenting a solution or presenting a solution that has nothing to do with the problematic situation.	Not presenting a solution or presenting a solution that has nothing to do with the problematic situation.
	L_2	Using the multiplication operation or a formal repetition of additions or verbal, symbolic, or visual representations shows many errors or difficulties in completing the task.	Demonstrates that he intends to do multiplication, although he makes many mistakes.	Presenting a partially inadequate response to the problematic situation.
	L_3	Using the multiplication operation or a formal repetition of additions or verbal, symbolic, or visual representations shows some errors or difficulties in completing the task.	Performing multiplication, although with some errors.	Respond appropriately the problematic situation, even if the result is incorrect.
	L_4	Use the multiplication operation or verbal, symbolic, or visual representations that show an understanding of the additive meaning.	Perform multiplication correctly and get the correct product.	Respond appropriately to the problematic situation, identifying the value obtained and its meaning.
4	L_1	Not presenting a solution or presenting a solution that has nothing to do with the problematic situation.	Not presenting a solution or presenting a solution that has nothing to do with the problematic situation.	Not presenting a solution or presenting a solution that has nothing to do with the problematic situation.
	L_2	Using the multiplication operation or verbal, symbolic, or visual representations shows many errors or difficulties completing the task.	Demonstrates that he intends to do multiplication, although he makes many mistakes.	Presenting a partially inadequate response to the problematic situation.
	L_3	Using the multiplication operation or verbal, symbolic, or visual representations shows some errors or difficulties in completing the task.	Performing multiplication, although with some errors.	Respond appropriately to the problematic situation, even if the result is incorrect.
	L_4	Use the multiplication operation or verbal, symbolic, or visual representations that show an understanding of the combinatorial meaning.	Perform multiplication correctly and get the correct product.	Respond appropriately to the problematic situation, identifying the value obtained and its meaning.

The research team developed both the initial and final tasks, ensuring their similarity to the tasks investigated in the Multiplication applet.

We utilized the assessment of tasks to analyze the level of knowledge (LK), global level of knowledge (GLK), and global performance (GP) of the students. The evaluation of initial tasks allowed for the identification of optimal levels of discrepancy among students, which were then grouped into 12 pairs according to the conditions of the zone of proximal development (Vygotsky & Cole, 1978). As part of this research, initial and final tasks were assessed using rubrics (Andrade, 2000), incorporating overarching criteria outlined in **Table 1**.

Table 2 illustrates the precise descriptions tailored to the mathematical content inherent in the tasks. The general criteria and specific descriptions were developed by the research team based on Andrade (2000), and Pratas et al. (2016) and were validated by three doctoral-level teachers. The specific descriptions focused on three specific objectives: Understanding the additive meaning of multiplication (objective 1), performing multiplication (objective 2), and identifying the result of the operation in the context of the task (objective 3) (**Table 2**). These objectives aimed to enhance students' mathematical knowledge regarding the meanings of the arithmetic operation of multiplication that were expected to be achieved.

Table 3. Percentage values assigned to LK for each specific objective of each task

Task	P (%)	LK	Objective 1	Objective 2	Objective 3
1	15	1	0%	0%	0%
		2	1%	2%	1%
		3	3%	5%	3%
		4	4%	7%	4%
2	20	1	0%	0%	0%
		2	1%	3%	1%
		3	4%	7%	4%
		4	5%	10%	5%
3	30	1	0%	0%	0%
		2	2%	4%	2%
		3	6%	10%	6%
		4	8%	14%	8%
4	35	1	0%	0%	0%
		2	3%	4%	3%
		3	7%	11%	7%
		4	10%	15%	10%

Note. P: Percentage

The LK was defined by calculating the median for each task, and the GLK was calculated using the median of the medians for each task. For GP, the specific descriptors mentioned above were maintained and a percentage value was assigned to each task. The final value of each student's performance was obtained by adding the various percentage values obtained in each task.

The interest in using rubrics stems from the idea that they are effective in assessing students' performance. Their effectiveness is justified by the use of specific criteria, which increase the reliability of the assessment (Panadero et al., 2023). The use of rubrics aimed to identify students' difficulties in supporting learning, meaning contributing to the construction of an action plan based on the identified challenges (Brookhart & Chen, 2015; Petkov & Petkova, 2006). The assessment of initial and final tasks using rubrics also aimed to analyze students' progress before and after the intervention (Petkov & Petkova, 2006). Based on the levels assigned to each specific objective in each task, the median was calculated for each task to obtain the LK of the students per task. Then, the median of the medians of each task was calculated to obtain the GLK of the students. The GP of the students was obtained based on the percentage values assigned to each specific objective in each task (**Table 3**). Firstly, a percentage value was assigned to each LK for each specific objective of each task. Then, the various percentage values obtained in each task were added together. Finally, the final values were organized into classes: [0; 25[, [25; 50[, [50; 75[, and [75; 100]. **Table 3** illustrates the percentage values assigned to each LK for each specific objective of each task.

Throughout the five sessions, we gathered written records answered by students, written records generated by their peers, audio recordings, screen recordings of the computers where the students utilized FlashBack Express software, and photographs. Data collection for the construction of multimodal narratives followed the protocol presented by (Lopes et al., 2014). A multimodal narrative describes the actions of the teacher and students during a lesson or task, fulfilling a set of established characteristics (Lopes et al., 2014).

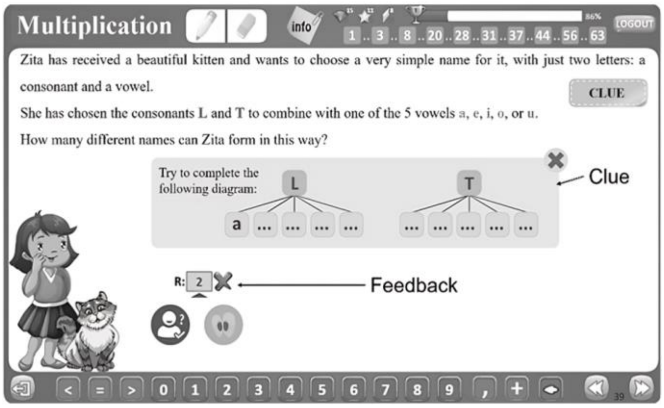


Figure 3. Immediate feedback of applet (Source: Authors' own elaboration)

Pedagogical Intervention

The pedagogical intervention phase consisted of five sessions aimed at understanding the meanings of the arithmetic operation of multiplication through the use of an applet. During these sessions, students (in pairs) solved problem situations using the Multiplication applet from the Hypatiamat Platform (https://www.hypatiamat.com/). This applet contains frames with problem situations and explanatory frames related to the arithmetic operation of multiplication. Its intuitive feature and the ability to provide immediate feedback and give hints (**Figure 3**). enable students to be active and autonomous during its usage (Pinto et al., 2022).

The five sessions of the intervention phase followed a similar structure. Each session was structured into four phases, aligning with the characteristics of the ET model proposed by Canavarro et al. (2012): introduction of the task (phase 1); development of the task (phase 2); discussion of the task (phase 3); and systematization of mathematical learning (phase 4). With this structuring, we aimed to contribute a form of intervention that integrates digital artifacts into the classroom to promote student learning (Clark-Wilson et al., 2020). Before each session, the trained teacher writes on the board the information communicated in phase 1, such as the learning objective, the tasks to be explored, the time allocated for each phase of the lesson, and other information, such as "answer first in the exploration guide paper before validating the answer in the applet", "explain how you thought in the exploration guide", and "read the statement carefully". This information remained on the board during phase 1 and phase 2; therefore, the students could recall the tasks (Canavarro, 2011). The workspace for the pairs of students in the

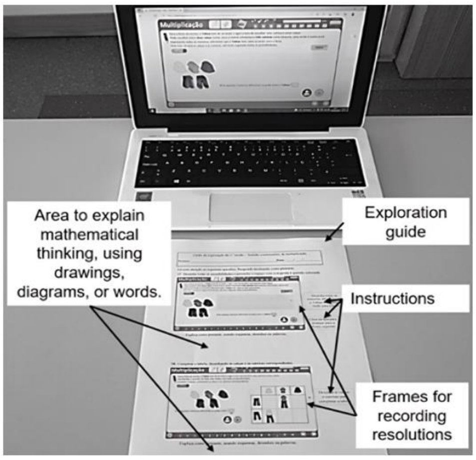


Figure 4. Laptop and an exploration guide paper (Source: Authors' own elaboration)

classroom was also organized before the lesson. On each pair's table, a laptop and an exploration guide paper were placed (**Figure 4**). The exploration guides were constructed by the research team of this study and validated by the research team of the Hypatiamat project. Each exploration guide contained the frames specific to the session, an area to explain mathematical reasoning (through drawings, diagrams, or words), and instructions to teach students during the manipulation of the applet (**Figure 4**).

The utilization of the exploration guide is in line with the recommendations of Lopes and Costa (2019). They advocate that for a digital artifact to yield educational advantages, it's crucial to accompany it with other epistemic artifacts, such as an exploration guide. This guide should provide clear instructions and enhance the utilization of digital artifacts. Before each session, the TT ensured that each pair's computer remained connected to the applet. These preparations were made to streamline the use of class time efficiently. Below is the outline of each session.

In phase 1 of each session, the TT commenced by organizing the pairs at their respective workstations. Subsequently, she conveyed the session's objective and introduced the tasks that students were expected to undertake, ensuring clarity and understanding among all pairs. Moving into phase 2, the pairs engaged in solving tasks using the Multiplication applet and the exploration guide. The frames explored contained problem situations (**Figure 5**) designed to elicit mathematical ideas pertinent to the concept of multiplication. These ideas were then shared and discussed in the subsequent phase. Consistent with the recommendations of Canavarro et al. (2012), the presented tasks were crafted to focus on problems necessitating interpretation (**Figure 5**).

Additionally, frames with explanatory purposes (**Figure 6**) were utilized, empowering students to navigate the learning process autonomously concerning the involved concepts.

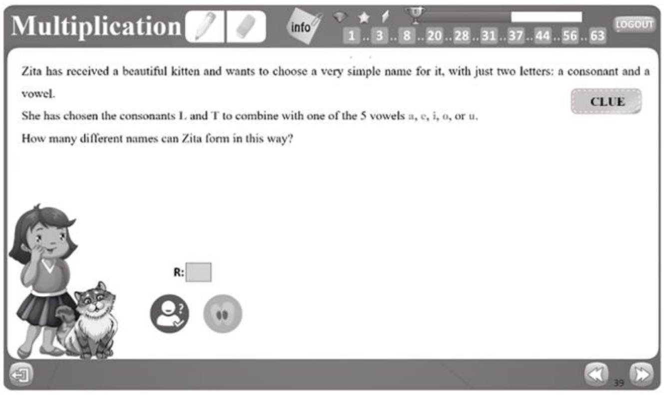


Figure 5. A sample frame to explain the problem (Source: Authors' own elaboration)

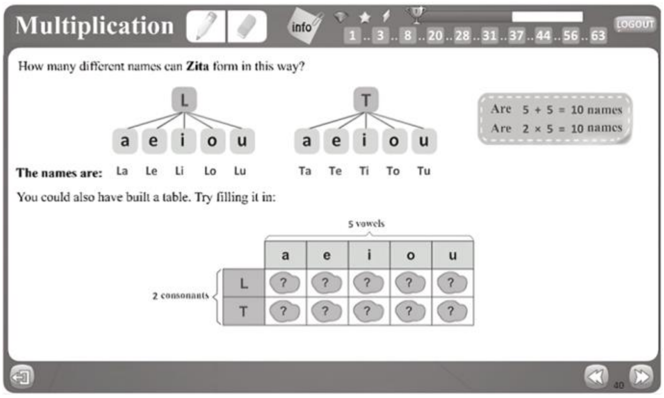
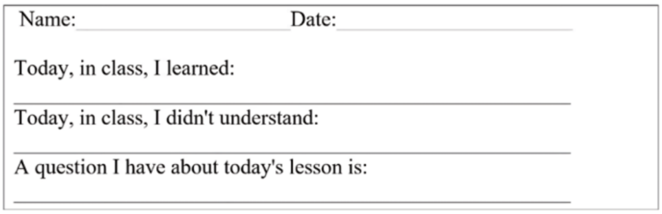


Figure 6. A sample frame for the solution of the problem (Source: Authors' own elaboration)

These frames also served as discussion prompts in the ensuing phase, aiding in the comprehension of multiplication concepts. Throughout this phase, the TT guided students through their difficulties via strategic questioning, as advocated by Canavarro (2011). Furthermore, the TT selected pairs' solutions for sharing and discussion in phase 3. These selections were made in accordance with the criteria outlined by Canavarro et al. (2012), targeting solutions with significant errors for exploration or representations pivotal to understanding multiplication concepts. These solutions were documented through photography for later projection onto the interactive board.

In phase 3, the selected solutions were projected on the interactive board, shared by their respective pairs, and then discussed with the class, mediated by the trained teacher. This mediation was carried out through questions such as "Do you agree?" or "Why?" which served to encourage discussion (Guerreiro et al., 2015). In phase 4, a synthesis sheet was distributed to each student to review the meaning of multiplication discussed during the session and to summarize the main aspects related to that meaning. The TT read the information and questions on the synthesis sheet and selected students to respond. Each student was responsible for recording the synthesized learnings on their sheet. Assessment is a regulatory process that should be part of the student's learning process.


Name: Date:

Today, in class, I learned:

Today, in class, I didn't understand:

A question I have about today's lesson is:

Figure 7. A formative assessment task "exit tickets" (Source: Authors' own elaboration)

Formative assessment is recognized as a type of assessment focused on students' learning and augmenting their knowledge (Lopes & Silva, 2020). In this context, even during phase 4, students completed the formative assessment task "exit tickets" (**Figure 7**), thereby facilitating a moment of self-assessment (Lopes & Silva, 2020).

These self-assessment moments required a redefinition of the planned activities (Amado, 2017), particularly concerning the incorporation of opportunities to address students' inquiries during the discussion phase. To facilitate this, PowerPoint presentations created by the research team were utilized to aid in clarifying doubts and providing additional support as needed.

Data Analysis

The statistical analysis was conducted using descriptive statistics, focusing on the description of students' GLK and GP obtained in the preintervention and post-intervention phases. The characterization of GLK and GP was performed using frequency tables, mean (*M*), and standard deviation (*SD*). Similarly, for students' GP, the intervals [50; 75[and [75; 100] were considered positive, while the intervals [0; 25[and [25; 50[were considered negative. The statistical analysis was conducted using descriptive statistics, focusing on the description of students' GLK and GP obtained in the pre- and post-intervention phases. The characterization of GLK and GP was performed using frequency tables, *M*, and *SD*. Similarly, for students' GP, the intervals [50; 75[and [75; 100] were considered positive, while the intervals [0; 25[and [25; 50[were considered negative.

The paired-sample t-test was utilized to compare the GLK and GP obtained in both the pre- and post-intervention phases, following the validation of its assumptions (Marôco, 2021; Pallant, 2020). The normality assumption for each of the dependent variables was evaluated using the Shapiro-Wilk test (Marôco, 2021; Pallant, 2020). In instances where normality was not satisfied, symmetry analysis was performed under the following conditions (Pallant, 2020; Pestana & Gageiro, 2020):

$$\left|\frac{skewness\ coefficient}{error\ of\ the\ sckewness\ coefficient}\right| \leq 1.96. \qquad (1)$$

The effect size value for the paired-samples t-test is calculated using Cohen et al.'s (2017) *d*, and the effect

Table 4. Pre- and post-intervention LK pre-intervention

LK	L_1	L_2	L_3	L_4
	Negative		Positive	
	Pre-intervention			
LK T1	70.8% (17)	0.0% (0)	12.5% (3)	16.7% (4)
LK T2	62.5% (15)	0.0% (0)	12.5% (3)	25.0% (6)
LK T3	54.2% (13)	4.2% (1)	8.3% (2)	33.3% (8)
LK T4	37.5% (9)	50.0% (12)	0.0% (0)	12.5% (3)
GLK	70.8% (17)	8.3% (2)	8.3% (2)	12.5% (3)
	Post-intervention			
LK T1	0.0% (0)	8.3% (2)	20.8% (5)	70.8% (17)
LK T2	8.3% (2)	12.5% (3)	8.3% (2)	70.8% (17)
LK T3	20.8% (5)	16.7% (4)	16.7% (4)	45.8% (11)
LK T4	0.0% (0)	50.0% (12)	16.7% (4)	33.3% (8)
GLK	8.3% (2)	12.5% (3)	41.7% (10)	37.5% (9)

size is classified, as follows (Marôco, 2021; Pallant, 2020): small ($d \leq 0.2$), medium ($0.2 < d \leq 0.5$), large ($0.5 < d \leq 0.8$), and very large ($d > 0.8$). All statistical analyses were performed using IBM SPSS statistics software (version 28, IBM USA), with a significance level of 5%.

RESULTS

Quantitative Results

Levels of knowledge

Table 4 shows an evolution in students' LK across all tasks between the pre- and post-intervention phases. The positive levels are 3 and 4 and the negative levels are 1 and 2. **Table 4** shows that, in task 1 (T1), more than 70% of the students exhibited a negative level (level 1) in the pre-intervention phase. After the intervention, more than 90% of the students were at level 3 and level 4. In task 2 (T2), it is evident that, in the preintervention phase, over 60% of the students were in level 1 and level 2. After the intervention, there is a positive evolution, with over 70% of the students in level 3 and level 4. In task 3 (T3), it can be seen that, before the intervention, over 50% of the students were in level 1 and level 2. After the intervention, there is an increase in level 3 and level 4, with over 60% of the students in these levels. In task 4 (T4), 80% of the students were in negative levels in the pre-intervention phase. In the post-intervention phase, 50% of the students were in level 3 and level 4, and 50% were in level 2. Regarding the GLK, the majority of students (79.1%) were in negative levels before the intervention. After the intervention, the majority (79.2%) shifted to positive levels.

From **Table 5**, we observe significant differences in the GLK of the students between the pre- and post-

Table 5. Descriptive statistics and pre- and post-intervention comparison at GLK

	M	SD	t	p	p
Pre-intervention	1.79	1.06	-7.00	0.001	1.518
Post-intervention	3.25	0.85			

Table 6. Frequency distribution of the DG

Class	Pre-intervention	Post-intervention
[0; 25[	54.2% (13)	0.0% (0)
[25; 50[	25.0% (6)	29.2% (7)
[50; 75[	16.7% (4)	20.8% (5)
[75; 100]	4.2% (1)	50.0% (12)

Table 7. Descriptive statistics and pre- and post-intervention comparison at GP

		M	SD	t	p	d
T1	Pre-intervention	3.54	4.51	-10.20	0.001	3.13
	Post-intervention	13.87	1.19			
T2	Pre-intervention	6.08	6.23	-10.75	0.001	2.57
	Post-intervention	17.91	1.92			
T3	Pre-intervention	10.92	9.96	-2.85	0.005	0.71
	Post-intervention	18.67	11.79			
T4	Pre-intervention	9.25	9.42	5.71	0.001	1.17
	Post-intervention	20.83	10.32			
GP	Pre-intervention	25.85	16.06	-11.46	0.001	2.11
	Post-intervention	68.21	23.49			

intervention phases. There was a significant increase between the mean value in the pre-intervention phase (1.79) and the post-intervention phase (3.25). Moreover, the t-value for the paired-samples t-test is -7.00, indicating a substantial and statistically significant difference between the pre- and post-intervention GLK scores. The p-value is 0.001, indicating that the observed difference is highly significant. The effect size (*d*) is calculated as 1.518, indicating a very large effect size according to Cohen et al.'s (2017) classification. This result indicates that the students' mean level in the pre-intervention phase was below two, and after the intervention, it was above three. This demonstrates that, according to the established criteria, the students showed significant improvements in understanding the meanings of the arithmetic operation of multiplication.

Global performance

Table 6 shows that there are improvements in the overall performance of the students. In the pre-intervention phase, more than 70% of the students had a negative performance, with 13 students scoring below 25%. In the post-intervention phase, a significant improvement is observed, with more than 70% of the students achieving a performance superior to 50%. It is also noteworthy that none of the students scored below 25%, although seven students still showed a negative performance.

Table 7 presents statistically significant differences in the mean performance results of the students across all tasks. The positive evolution is evident in the strategies used by the students in solving tasks during the pre- and post-intervention phases, as presented below. With the same purpose, excerpts from the multimodal narratives will be presented, demonstrating in detail the students'

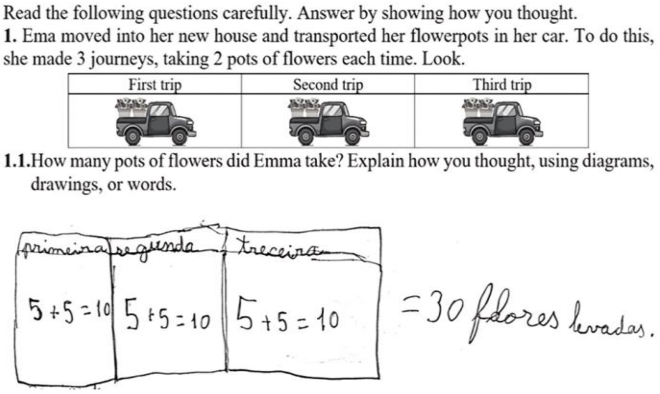


Figure 8. Example solution student A's in pre-invention phase (Source: Authors' own elaboration)

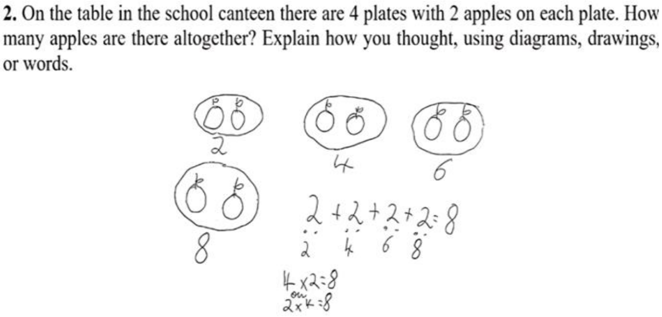


Figure 9. Example solution student M's in pre-invention phase (Source: Authors' own elaboration)

reasoning, the actions of the TT, and the moments of the class that may have contributed to this positive evolution. We begin by presenting some illustrative examples of the strategies employed by the students in each task, in the pre- and postintervention phases.

Qualitative Results

Pre-intervention phase: T1 and T2

Figure 8 and **Figure 9** depict the solutions for two exercises completed by student A and student M, respectively, prior to the intervention. The two exercises are related to the additive meaning of multiplication. The distinction between them lies in the fact that the first includes a visual representation of the problem, while the second does not.

In **Figure 8**, student A's solution fails to demonstrate any relation with the additive meaning of multiplication. The student did not provide any solution related to multiplication and incorrectly identified the result, indicating a proficiency level of 1 according to the classification criteria of the established knowledge level.

Contrarily, in **Figure 9**, student M's solution exhibits some inaccuracies. She considers that the operations 4 × 2 and 2 × 4 mean the same thing. The operation 2 × 4 does not correspond to the number of times the addends are repeated in addition. Despite these errors, her solution positions her at level 3 based on the classification criteria of the established knowledge level.

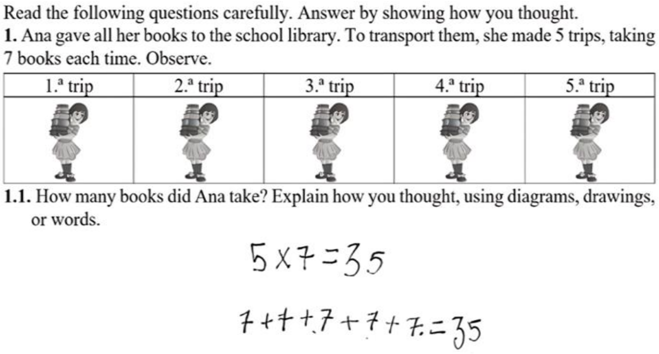


Figure 10. Example solution student A's in post-invention phase (Source: Authors' own elaboration)

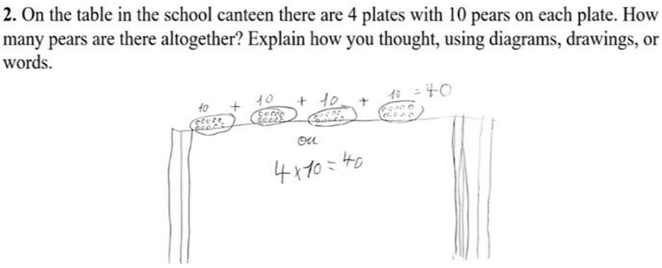


Figure 11. Example solution student M's in post-invention phase (Source: Authors' own elaboration)

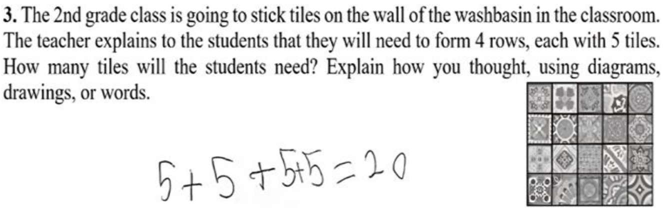


Figure 12. Example solution student G's in pre-invention phase (Source: Authors' own elaboration)

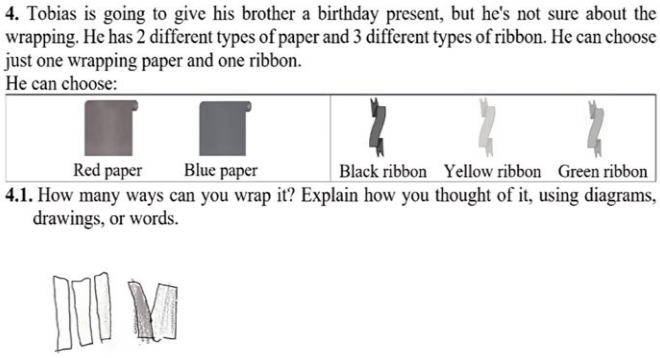


Figure 13. Example solution student I's in pre-invention phase (Source: Authors' own elaboration)

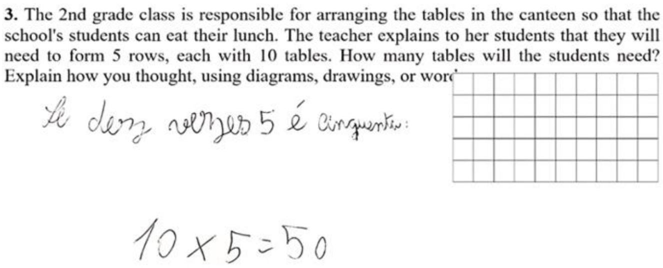


Figure 14. Example solution student G's in post-invention phase (Source: Authors' own elaboration)

Post-intervention phase: T1 and T2

Student A and student M (Figure 10 and Figure 11) have presented solutions that demonstrate a comprehensive grasp of the additive interpretation of multiplication, thereby attaining level 4 according to the established criteria for knowledge classification. These students effectively utilized the multiplication operation and arrived at the correct solution. Importantly, they recognized that the order of the factors in the multiplication operation corresponds to the number of repetitions of the addends in addition.

Pre-intervention phase: T3 and T4

Figure 12 and Figure 13 depict the solutions for two exercises completed by student G and student I, respectively, prior to the intervention. Exercise three is related to the rectangular configuration of the additive meaning of multiplication. Exercise four is related to the combinatory meaning of multiplication. Student G (Figure 12) did not provide any solution related to the operation of multiplication. His solution was based solely on addition. Student I (Figure 13) encountered difficulties in completing the task, indicating a lack of understanding of the combinatorial meaning. As a result, both students are classified at level 1 according to the established criteria for knowledge level classification.

Post-intervention phase: T3 and T4

The solution of student G (Figure 14) demonstrates a complete understanding of the additive meaning of multiplication, as she was able to use the multiplication operation and present the correct result. Her solution places her at level 4, according to the established criteria for classifying the LK. The solution of student I (Figure 15), although not completed, already shows some understanding of the combinatorial meaning of the arithmetic operation of multiplication, as it is possible to observe a combination (a crepe with a filling and a topping). This indicates that student I have reached level 3, according to the established criteria for classifying the LK.

Intervention phase

Aspects of the intervention related to the phases of ET are presented, which had a positive impact on student learning. During pair work, a characteristic of this teaching model, it was observed that students learned actively and collaboratively, as evidenced in the following excerpt:

Student L (group 11): Two apples?

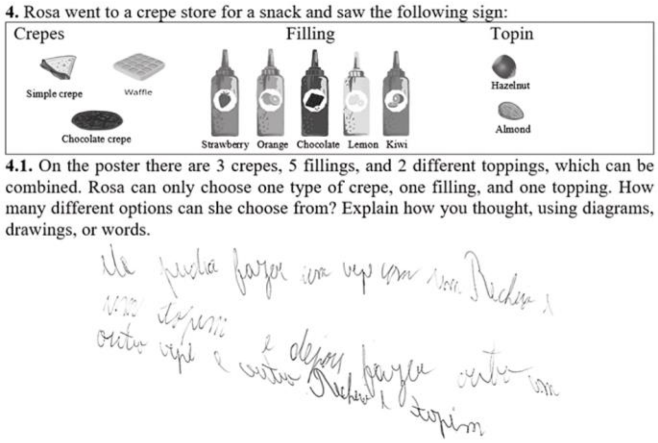


Figure 15. Example solution student I's in post-invention phase (Source: Authors' own elaboration)

Student R (group 11): No, one apple.

Student L (group 11): Huh? I thought one apple and one apple.

Student R (group 11): Wait. Um ... this milk with this bread (1-second pause) and a pear.

Student L (group 11): Lemon juice ... orange juice.

Student R (group 11): Orange juice with this bread.

Student L (group 11): I was going to say lemon.

Student R (group 11): Kiwi and this milk with this croissant. (the student was pointing to the possibilities on the exploration guide as she explained)

In the previous excerpt, the use of the exploration guide is visible. Student R points to the combinations listed in the exploration guide as she explains her mathematical reasoning. In some pairs, the TT needed to intervene to encourage collaboration:

TT: You have to work in a group. You can't have one on the computer and the other on the sheet, student N.

During the task development phase, the TT can be seen encouraging students to think about the meaning of the factors in the context of the task through questions:

Student I (group 9): We thought it was three times four, but it wasn't.

TT: It wasn't three times four.

Student I (group 9): It was four times three.

TT: And why was it four times three? (pause of 2 seconds). Look at the addition there (pause of 4 seconds).

Student I (group 9): Because here it's three plus three plus three plus three.

In the following excerpt, it is evident that the immediate feedback provided by the applet during this phase encouraged students to think and understand why their solution was incorrect.

Student K (group 10): Um ... so, we were wrong because it's like there were four baskets and not three baskets of four.

The applet proved to be intuitive for the students, which resulted in them not showing any difficulties during its manipulation. The exploration of explanatory frames, characteristic of the applet, during the development phase, allowed for active and autonomous learning, as evidenced below:

TT: But did you answer without looking or looking? (meaning to ask if you answered on the exploration guide with the support of explanatory frames 45 to 54, referring to the solution explanation)

Student X (group 6): Without looking because I thought it was the two bread times these (the drinks) and then since there were three apples ... (referring to the fruits).

TT: Right.

Student X (group 6): ... three fruits, I multiplied this (pointing to the multiplication operation recorded in the exploration guide), and it gave twelve.

The explanation provided by student X demonstrates an understanding of the meaning of the factors in the arithmetic multiplication operation within the context of the task. During the discussion moments, students showed difficulties in engaging in autonomous discussion. This necessitated the guidance of the TT through questions, as can be seen in the following excerpt:

TT: Equal to twelve (TT wrote the operation $4 + 4 + 4 = 12$ on the board, below the operation $3 \times 4 = 12$). Does this addition (pointing to $3 \times 4 = 12$) represent this addition? (pointing to $4 + 4 + 4 = 12$).

Some students: No.

TT: Student O, what do you say?

Student O (group 1): Oh ... no.

TT: So how is the addition of 4×3?

Student O (group 1): Oh ... (pause) it's four ...

TT: Four.

Student O (group 1): Plus four plus four.

TT: Plus four. But what you just said is the same as what's up here (pause of 2 seconds). Who wants to help their classmates? Student M (student M raised her hand).

Student M (group 5): It has to be ... three plus three, plus three.

During the discussion phase, we observed student I, participating in the combinatorial reasoning being constructed through the exploration of a tree diagram:

TT: It could be bread with juice or with milk (4-second pause while student I fill in the diagram as TT speaks). Now we have the bread, we have the juice, now we need some fruit (1-second pause). What about?

Student I (group 9): Apple?

TT: Apple. (5-second pause while student I writes "apple") Or?

Student I (group 9): Kiwi.

TT: (28-second pause while student I writes "kiwi" and TT talks to a student, letting them go to the bathroom) Or?

Student I (group 9): Pear.

TT: Pear. (5-second pause while student I writes "pear") Then the bread can go with milk. But then we need the fruits, right? (the student nods) Which ones?

Student I (group 9): Apple?

TT: Apple (7-second pause while student I writes "apple").

Student I (group 9): Or pear.

TT: Or pear (4-second pause while student I writes pear). Or?

Student I (group 9): Kiwi.

The moments of clarification of doubts served as an extension of the discussion phase, as discussions arose around the difficulties pointed out by the students during self-assessment. In the 3rd session, student S recorded in the formative assessment task that he had not understood the task present in frame 39 (**Figure 16**).

In the following excerpt, we see that the moment of clarification of doubts allowed for the clarification of student S's question:

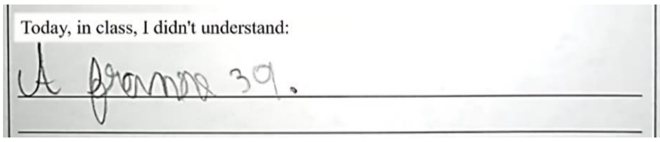


Figure 16. Difficulty pointed out by the student I (Source: Authors' own elaboration)

TT: Explain this frame to me.

Student H (group 9): Me?

TT: Yes.

Student H (group 9): Uh ... the girl ... the girl could give ten names to the cat.

TT: Ten?

Student H (group 9): Yes.

TT: So why ten? (2-second pause).

Student H (group 9): Because it's La, Le, Li, Lo, Lu, Ta, Te, Ti, To, Tu.

TT: why is it La, Le, Li, Lo, Lu, and Ta, Te, Ti, To, Tu? Come explain at the board, do you want to explain? (student H shook his head no). Student S.

Student S (group 3): I kinda understood.

TT: Then come explain, come (4-second pause as the student moves to the interactive board).

Student S (group 3): The La, Le, Li, Lo, Lu, we combine with these vowels (pointing to the vowels on the third slide projected on the interactive board), and the Ta, Te, Ti, To, Tu (again pointing to the vowels on the third slide projected on the interactive board).

TT: Did you hear what student S said?

Some students: Nooo!

TT: Then explain it louder.

Student S (group 3): I combine the L with the vowels and the T with the vowels too.

Figure 17 displays a section of the systematization sheet used for concluding the class.

The following excerpt demonstrates that student H accurately answered the task presented on the summarization sheet. The student successfully identified the meaning of the factors within the context of the task ("Four times two").

TT: What does the additive meaning of the arithmetic multiplication operation say? (2-

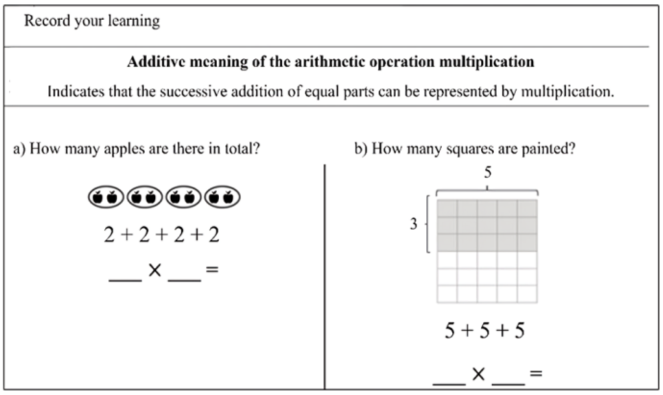


Figure 17. A section of the systematization sheet (Source: Authors' own elaboration)

second pause). Student H read it aloud (asking to read the definition of the additive meaning present on the summarization sheet projected on an interactive board).

Student H (group 9): I can't see it.

TT: Come here, read it on your sheet.

Student H (group 9): It indicates that the successive addition of equal parts can be represented by multiplication.

TT: Very good, it's in ... in item "a", we have this question: How many apples are there in total?

Student H (group 9): Eight.

TT: Eight. And how did you fill in the ... the multiplication?

Student H (group 9): Four times two.

DISCUSSION

This study aimed to analyze an exploratory intervention approach that integrates digital artifacts in the classroom to promote the understanding of mathematical concepts. The analysis of initial tasks revealed students' difficulties regarding the meanings of the arithmetic operation of multiplication operation, corroborating Kouba and Franklin (1995) and aligning with the challenges identified by Isoda and Olfos (2021), Ribeiro and Almeida (2022) and Ribeiro et al. (2023). The student's difficulties corroborated what Gómez (1991) says, that problems involving successive addition are easier for students and that when solving problems involving successive addition of equal installments, students find it difficult to distinguish the meaning of the factors in the context of the task. It was observed that students encountered difficulties in providing a solution that demonstrated comprehension of the meaning of multiplication within the context of the problem situation, as well as in performing the multiplication operation and identifying the result within the task context. The use of rubrics facilitated the analysis and identification of students' difficulties. This analysis led to the definition of an action plan aimed at overcoming students' challenges, aligning with Brookhart and Chen (2015) and Petkov and Petkova (2006). These difficulties were reflected in the results of LK, GLK, and GP of the students in the initial phase. The presented statistical results demonstrate a positive evolution in students' LK in all tasks after the intervention. This evolution was evident with a significant increase in the number of students at positive levels after the intervention, compared to the pre-intervention phase. Additionally, a statistically significant improvement was observed in students' GLK, indicating an enhancement in their understanding of the meanings of the arithmetic operation of multiplication. The results of the distribution of absolute and relative frequencies of the LK per task justify what Gómez (1991) refers to. Specifically, students encounter the most difficulties in task 4, understanding the combinatorial meaning of multiplication. This difficulty was more noticeable when the number of combinations increased, leading to a greater total of possible combinations. The results of descriptive statistics on students' GP revealed statistically significant differences, indicating a positive evolution in students' GP after the intervention.

The implementation of the ET model appears to have been fundamental for the observed improvements. The results related to the different phases suggest that integrating applets in the classroom following the ET model is an effective approach to promoting students' learning development regarding the meanings of multiplication. Its structured approach, aligned with the different phases proposed by Canavarro et al. (2012), facilitated the integration of the applet in the classroom, resulting in significant learning outcomes for students. The ease with which students manipulated the applet allows us to agree with the idea that it is necessary to select the most suitable digital artifact for students (Gorev & Gurevich-Leibman, 2015). The suitability of the digital artifact used, and its inclusion also aligns with Martins (2020) perspective on structured and meaningful utilization.

The collaborative work in pairs, characteristic of this teaching model, combined with the immediate feedback provided by the applet during activities, encouraged active and collaborative learning. The immediate feedback helped the students understand the problem through visualization, agreeing with Gorev and Gurevich-Leibman (2015). The active use of the applet allowed students to take ownership of their learning, collaborate with their classmates, become autonomous, and participate in debates, as mentioned by Jahnke et al. (2022) and Office of Educational Technology (2024).

The improvements observed are likely to have resulted, in part, from the interaction of students with the applet. This aligns with the idea that using digital artifacts can provide a favorable interaction between students and digital tools conducive to understanding mathematical content, as suggested by Long and Bouck (2023) and Martins et al. (2019). The use of the applet favored the learning of the mathematical content, corroborating the studies by Pelton et al. (2018) and Zhang et al. (2015). However, it is worth highlighting the importance of the guidance provided by the TT during the use of the applet. Her guidance directed students toward understanding the meanings of multiplication arithmetic operation factors. During the task development phase, it is evident that the use of the applet and the TT's mediation encouraged students to think about the meaning of factors in the context of the task. This leads us to concur that the utilization of the applet as an epistemic tool, along with the epistemic mediation conducted by the TT, contributed to facilitating the learning of the meanings of the arithmetic operation of multiplication (Costa et al., 2021; Lopes & Costa, 2019; Silva, 2021).

The ET allowed students to participate actively, share their solutions, and reflect on mathematical knowledge, such as seen in Ponte and Quaresma (2020). The participation of student I in the construction of combinatorial reasoning during the discussion highlights the importance of discussion moments in knowledge construction. The discussion moments provided an opportunity to address difficulties and build knowledge actively (Ferreira & Ponte, 2017; Huang & Sutherland, 2022; Ponte, 2017). However, the results show that discussion did not arise spontaneously. There was a need to encourage students to discuss, as noted by Guerreiro et al. (2015). The phase of systematization of mathematical learning provided feedback to the TT about the learning achieved by the students but also allowed them to reinforce aspects related to the meanings of the arithmetic operation of multiplication (Canavarro, 2011; Canavarro et al., 2012). Formative assessment in this phase also played an important role during the intervention. The utilization of formative assessment tasks facilitated the adaptation of practice based on students' difficulties, including opportunities for clarification of doubts. Implementing formative assessment, particularly through self-assessment instruments, appeared to have a significant effect on student learning, contributing to the enhancement of students' knowledge (Lopes & Silva, 2020).

It is also important to highlight that the use of the exploration guide supported student learning, enriching the use of the applet, consistent with what Lopes and Costa (2019) mentioned. This was evident when student R used it to support the explanation of her mathematical reasoning.

In summary, the results suggest that this form of intervention facilitated students' understanding of the meanings of multiplication through active and meaningful participation, corroborating with Sedaghatjou and Rodney (2018).

CONCLUSION

The results demonstrated that integrating the applet in the classroom, based on ET, significantly contributed to promoting the learning of the meanings of the arithmetic operation of multiplication among 2nd grade students in elementary school. Moreover, this study showed that ET allows for a more structured integration of digital artifacts and the development of essential skills such as autonomy and collaboration. Thus, we address the need to understand how teaching and learning unfold in a technology-rich environment. This exploratory intervention approach allowed students to construct their knowledge actively and meaningfully. The phases of this ET model significantly contributed to student learning. The autonomy granted to students during task development with the applet and the mediation provided by the TT benefited student learning. The exploration guide played an essential role in supporting student autonomy through clear instructions. Moments of discussion and clarification of doubts raised by students in self-assessment were essential for knowledge construction. Similarly, we highlight the importance of formative assessment, demonstrating how the use of formative assessment tasks and rubrics can assist teachers in assessing student knowledge and performance. We provide a way to identify and analyze student difficulties, aiming to promote more personalized teaching adapted to individual student needs. We acknowledge several limitations in this study, including sample characteristics, the digital artifact used, and the mathematical concepts explored. The study focused solely on 2nd grade students. We suggest that future studies explore this integration of digital artifacts in other grade levels to identify aspects that could contribute to more suitable integration across different grade levels. Additionally, integrating other digital artifacts could help understand if this integration method indeed facilitates integration effectively. Lastly, a more prolonged and in-depth exploration of the combinatorial meaning of the multiplication operation is suggested, as this was where the most difficulties were observed.

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Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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