MODESTUM EURASIA Journal of Mathematics, Science and Technology Education, 2024, 20(12), em2551
ISSN:1305-8223 (online)
OPEN ACCESS Research Paper https://doi.org/10.29333/ejmste/15707

Examining tenth-grade students' errors in applying Polya's problem-solving approach to Pythagorean theorem

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Received 22 July 2024 ▪ Accepted 28 October 2024

Abstract
This study uses Polya's problem-solving strategy to explore student errors and causal factors in Pythagorean theorem problem-solving. The sample was drawn from tenth-grade students at Al Hosn Secondary School in Abu Dhabi, United Arab Emirates in the academic year 2023/2024. Data were collected using two methods: written examinations, as outlined in Polya's strategy, and interviews with students who committed errors. The research test instrument consists of 3 trigonometric problem-solving. From 30 students of Al Hosn Secondary School, there were 48.9% of data errors, 50% of concept errors, 57.8% of strategy errors, 47.8% of calculation errors, and 14.4% of careless errors. The errors made by the students originated from their inability to comprehend the geometric interpretation of the Pythagorean theorem, difficulties in applying algebraic operations, and challenges in modelling the data provided in the problems.

Keywords: trigonometric, error, Pythagorean theorem, Polya's strategy, problem-solving

INTRODUCTION

Given the importance of problem-solving skills in mathematics education, this skill is the cornerstone of a student's overall development. Problem solving promotes critical thinking, encourages persistence, inspires curiosity, builds confidence, and helps students face real-life challenges (Castro & Herrera-Restrepo, 2024). Effective problem-solving requires systematic planning, logical thinking, and wise selection of strategies and implementation methods (BSNP, 2006).

Trigonometric problem-solving is a fundamental part of the mathematics curriculum in middle school. Students develop their abilities to analyze and solve mathematical and real-world challenges involving triangles and angles. It includes routine and non-routine problems and requires innovative thinking and the application of trigonometric principles. Teachers play a crucial role in developing students' problem-solving skills, emphasizing logical thinking, creativity, and practical application of triangular concepts. The importance of trigonometry extends beyond theoretical mathematics and has many applications in fields that require precise measurements and calculations. Therefore, developing effective trigonometry problem-solving strategies is critical for students' academic and career pursuits.

According to Barlow et al. (2018), errors in mathematics learning refer to mistakes made when solving mathematical problems using various methods, whether algorithms or procedures. Although teachers are familiar with some errors, they generally have no explicit obligation to incorporate student errors into the teaching process. However, viewing student errors as an essential skill can be helpful, especially when introducing new topics. Unfortunately, using error analysis as a teaching strategy in mathematics education remains common. Traditional teaching methods often rely on teachers to provide students with correct examples to imitate and provide students with limited opportunities to practice independent problem-solving and develop strategies.

This study was developed within the framework of the projects PID2021-127104NB-I00 (MICINN, FEDER, EU), FIED21-002 (SENACYT) and SGR_2021-00159, Generalitat de Catalunya.

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Contribution to the literature

- This study explores a significant contribution to the educational and mathematical literature. It provides a deeper understanding of students' common errors when applying the Pythagorean theorem, which helps teachers identify and correct these errors effectively.
- The study evaluates the effectiveness of the Pólya strategy in improving mathematical problem-solving skills, which supports the development of more efficient teaching methods appropriate to students' educational needs.
- The study directs attention to other areas in need of further research, such as studying the effect of various teaching strategies on students' understanding of mathematical concepts, making it a useful and important reference for future research in this area.

Students' obstacles in solving trigonometry problems can be attributed to conceptual misunderstandings, principal errors, and algorithm errors. The leading cause of these errors is students' lack of understanding of trigonometry concepts (Antara et al. 2020). Errors in math problems can manifest in many ways, including misunderstandings of symbols, misjudgments of place values, incorrect procedural steps, and even illegible handwriting. The analysis of students' learning difficulties plays a pivotal role in the enhancement of mathematics education. By identifying the specific barriers that hinder students' learning processes, educators can formulate targeted strategies aimed at overcoming these challenges. This approach enables teachers to directly address the root causes of students' struggles, thereby significantly improving their comprehension and mastery of mathematical concepts.

In the context of various studies examining the classification of errors made by students when applying the Pythagorean theorem, researchers have explored specific recurring error types. Some of these studies focused on errors, such as confusion between the concepts of hypotenuse and side or inaccuracies in calculating the side length of a triangle. Simultaneously, other studies delved into analyzing broader error classifications, including data, concept, strategy, calculation, and careless errors (Fahrudin et al., 2019; Veloo et al., 2015).

Our study aim was threefold:

(1) to pinpoint any errors within their responses,

(2) to categorize these errors by established research findings to ascertain their consistency with existing typologies or to identify novel deviations, and

(3) to elucidate the underlying reasons prompting these errors amongst students.

This endeavor enables the formulation of recommendations for teacher training, facilitating the comprehension for both pre-service and in-service educators regarding the nature of student errors, their typologies, and potential remedies. The main research question for this study is explicitly stated: "How does Polya's (1971) problem-solving strategy help identify student errors and their causal factors in solving Pythagorean theorem problems among tenth-grade students at Al Hosn Secondary School in Abu Dhabi, United Arab Emirates?"

This research highlights the need for tailored teaching approaches considering students' challenges. Barlow et al. (2018) emphasized careful selection and utilization of student errors for whole-class discussion. Addressing student errors and misunderstandings in the classroom can lead to a productive learning experience. Teachers who use these errors as teaching opportunities can promote a deeper understanding of mathematical concepts, benefiting all students' learning. This approach fosters an inclusive and participatory teaching environment.

We seek to determine whether the findings of this investigation align with those of earlier research. Furthermore, we anticipate the refinement of a more intricate classification of student errors, intending to deliver a valuable methodological resource for the scientific community. This tool will enable a more in-depth exploration of students' errors while addressing Pythagorean problems, fostering a deeper understanding of these practical processes. Consistent with prevailing research findings, students' predominant errors in navigating Pythagorean theorem-related problems are intricately tied to the precise modelling of the problem and the meticulous verification of the solution process. Some students need help differentiating between types of triangles and correctly identifying the sides of the triangle. Students interact with algorithms as they engage in problem-solving tasks, underscoring the necessity for precisely comprehending the Pythagorean theorem concept. This involves correctly establishing the relationships between the hypotenuse and the legs in a right-angled triangle.

THEORETICAL FRAMEWORK

Problem-Solving Approach

Enhancing proficiency in solving mathematical problems is a crucial objective of mathematics education, as it is considered a fundamental skill in the learning process. Mastering mathematical problem-solving skills

(De Almeida & de Castro, 2023) is essential and significant for students. According to Rizka and Lismalinda (2021), problem-solving stands at the core of the teaching and learning process, representing a primary skill in mathematical learning activities.

Polya's (1971) problem-solving steps serve as an approach to addressing mathematical problems. Hasan et al. (2019a, 2019b) emphasized diverse challenges in mathematics, including those associated with the Pythagorean theorem, encompassing conceptual, procedural, and computational errors. It is common for students to undertake problem-solving without adhering to prescribed stages, overlooking the pivotal step of recording known information and understanding the problem's specifications. The omission of documenting known information frequently results in errors when inputting values, potentially giving rise to conceptual errors. Students must receive correct evaluations to solve the assigned problems in the assessment phase.

Polya (1971) outlines four fundamental steps students should follow when addressing problems. These steps include:

(1) understanding the problem, which involves understanding the known and unknown data associated with the problem,

(2) devising a solution plan aimed at establishing the relationship between the known and unknown data to formulate a resolution strategy,

(3) executing the devised plan, involving the implementation of the planned steps to solve the problem by the devised strategy, and

(4) reviewing the solution, which entails examining the obtained solution and considering whether the resolution steps or solutions apply to other problems (Pratikno & Retnowati, 2018).

Figure 1 illustrates Polya's (1971) problem-solving strategy steps in sequence.

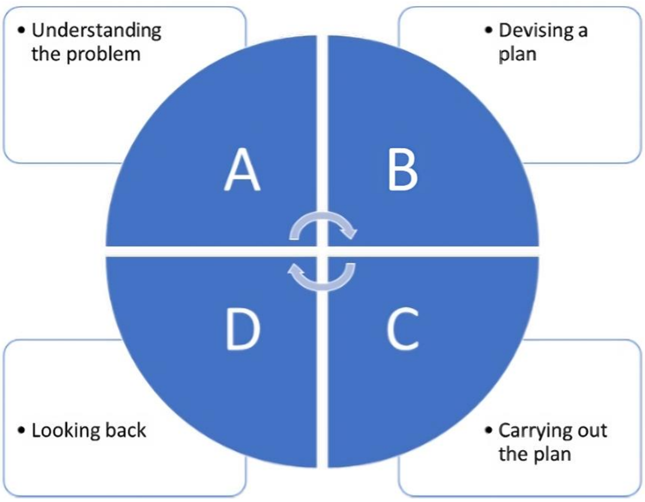

Figure 1. Flowchart steps for Polya's (1971) problem-solving

Types of Students' Errors in the Pythagorean Theorem

Previous scientific literature suggests that trigonometry is a branch of mathematics often considered challenging for middle school students to comprehend. Among the various mathematical concepts within trigonometry, one that is typically deemed fundamental in most curricula worldwide is the Pythagorean theorem. This theorem is frequently introduced in problem-solving, serving as a critical standard to be developed during secondary education (Hutapea et al., 2015). However, research also highlights students' difficulties when engaging with this theorem, particularly in the errors made when solving problems that rely on applying the Pythagorean theorem.

According to the research conducted by Rudi et al. (2020), who investigated school misconceptions about the Pythagorean theorem, the findings indicated that students need help to grasp the definitions, explain symbols or notations associated with mathematical concepts, and interpret mathematical objects. On the other hand, when solving problems related to applying the Pythagorean theorem, students demonstrated proficiency in describing procedures, algorithms, and techniques for addressing questions. Nurmeidina and Rafidiyah (2019) conducted a study examining students' difficulties in solving trigonometry problems. The results show that the students have difficulty understanding the information given to solve the problems. They make many errors in applying trigonometric concepts to answer the questions because they do not correctly calculate the results of angle comparison. Besides, they incorrectly determine the angle of contrast between the angles obtained.

According to Arivina and Jailani (2022), students need help interpreting language, misusing data, or distorting theory definitions, mainly when dealing with the Pythagorean theorem. Ahmad et al. (2018) proposed a more inclusive categorization of student errors related to the Pythagorean theorem, identifying five types of errors: failure of process skills, carelessness or inaccuracies, misunderstanding of problems, errors in the use of notation, and misconceptions of concepts. This classification is consistent with the findings of Veloo et al. (2015).

A research investigation by Sulistyorini (2018) examined errors in solving geometric problems among pseudo-thinking students, revealing several challenges. These identified errors encompassed misunderstandings in measuring a line segment, an inability to recognize that triangles should be right-angled, and errors in applying the Pythagorean theorem and trigonometric ratios. Moreover, students exhibited challenges in employing triangle congruence to substantiate the congruence of measurements for two angles.

Table 1. Analysis of the previous literature on students' errors

Previous research	Type of errors
Hanggara et al. (2024)	Conceptual error, procedural error, & principal error
Sekgoma and Salani (2023)	Calculations error, misidentification error, & interpretation error
Arivina and Jailani (2022)	Misinterpret language, misuse data, & distort the theorem definition
Setiawan (2022)	Misconception error, miscalculation error, & fact errors
Supardi et al. (2021)	Word use error, visual mediator error, narrative error, & routine error
Fahruding and Pramudya (2019)	Data error, conceptual error, strategy error, calculation error, & conclusion error
Hutapea et al. (2015), Sari and Wutsqa (2019), and Zulyanty and Mardia (2022)	Decoding, comprehension, transformation, process skill, & encoding
Veloo et al. (2015)	Conceptual error, careless error, problem-solving error, & value error

Several studies have delved into a more detailed and comprehensive classification of student errors associated with the Pythagorean theorem. For instance, Fahrudin et al. (2019) identified five types of student errors: data errors, concept errors, strategic errors, calculation errors, and conclusion errors. Similarly, Supardi et al. (2021) identified four misconception types: word use error, visual mediator error, narrative error, and routine error. In the same vein, scholars have employed the Newman error analysis classification to classify some of the errors made by students. This classification distinguishes errors in reading, comprehension, transformation, process skills, and encoding (Hutapea et al., 2015; Sari & Wutsqa, 2019; Zulyanty & Mardia, 2022).

In line with most of these research findings, students' primary errors when tackling problems associated with the Pythagorean theorem are linked to accurately modelling the problem (Hidayati, 2020; Moradi et al., 2023; Rohimah & Prabawanto, 2020; Wardhani & Argaswari, 2022), and verifying the solution process (Puspitarani & Retnawati, 2020; Satriani et al., 2020). Some students need help differentiating between types of triangles and correctly identifying the sides of the triangle. Students interact with algorithms as they engage in problem-solving tasks, underscoring the necessity for precisely comprehending the Pythagorean theorem concept. This involves correctly establishing the relationships between the hypotenuse and the legs in a right-angled triangle.

To summarize the main contributions of this review, we compiled a list of errors. Most of them have similarities. In **Table 1**, we collected all the main error types that emerged when reviewing previous studies.

Drawing on the key contributions of prior research, we developed our taxonomy, categorizing errors into five distinct types: data errors, conceptual errors, strategic errors, calculation errors, and careless errors. **Table 2** summarizes these five error types, based on the definitions we propose for each, in alignment with existing literature.

METHODS

This study is a qualitative approach. This approach was chosen to transcend the mere description of the outcomes attained by students when tackling the set of Pythagorean problems provided to them.

The responses garnered from the written text were systematically scrutinized against predetermined categories. Any errors in the students' submissions were meticulously identified and classified using the established categories within our classification system. Subsequently, these responses served as a basis for conducting interviews with the students, enabling a more thorough exploration of their answers and justifications to elucidate potential rationales underlying any errors. Employing discourse analysis techniques as outlined by Gee (2014), the responses proffered by the students were thoroughly examined. This analytical approach was complemented by additional teacher interviews, facilitating a comparative analysis to corroborate and validate the researchers' interpretations.

Setting and Participants

The study sample consisted of 30 students from Al Hosn Secondary School in the Emirate of Abu Dhabi in the United Arab Emirates. This study has been carried out during the academic year 2023-2024.

Data Collection Techniques

Two tools were applied in this study:

1. A written test consisting of three problem-solving tasks in trigonometry according to Polya's (1971) strategy, conducted by three teachers from Al Hosn Secondary School who teach the tenth grade.
2. A student-teacher interview guide related to the mistakes made by students while working on solving problems based on Polya's (1971) strategy.

The test items specified in the written test contain the four elements of mathematical problem-solving (understanding the problem, devising a plan, executing the plan, and carrying out the plan). Each student was allocated a 30-minutes to complete the test items declared valid on a pilot sample. During the assessment, one point is awarded for completing each of the four steps of Polya's (1971) problem-solving method to arrive at the correct solution. For example, students who successfully implemented the initial phase of Polya's

Table 2. Taxonomy of students' errors regarding the Pythagorean theorem

No	Error	Description	Alignment with the previous literature	Source/reference
1	Data	Students are writing down existing problem data wrongly Students are translating existing problems wrongly.	Misinterpret language, misuse data, word use error, visual mediator error, data error, & decoding	Arivina and Jailani (2022), Durmuş (2019), Fahrudin and Pramudya (2019), Hutapea et al. (2015), Sari and Wutsqa (2019), Supardi et al. (2021), & Zulyanty and Mardia (2022)
2	Concept	Students are determining the formula or theorem or definition to answer the problem incorrectly Students do not write formulas or theorems or definitions to answer problems.	Conceptual error, principal error, misidentification error, narrative error, comprehension, & transformation	Arivina and Jailani (2022), Durmuş (2019), Fahrudin and Pramudya (2019), Hanggara et al. (2024), Hutapea et al. (2015), Sari and Wutsqa (2019), Sekgoma (2023), Supardi et al. (2021), Veloo et al. (2015), & Zulyanty and Mardia (2022)
3	Strategy	Students try to operate at the right level on a problem but use procedures or methods that are not appropriate Students try to operate at the right level but choose inappropriate data information.	Procedural error, strategy error, & problem-solving error	Durmuş (2019), Fahrudin and Pramudya (2019), Hanggara et al. (2024), & Veloo et al. (2015)
4	Calculation	Students are giving or writing signs of mathematical operation incorrectly Students are counting mathematics operations such as adding, subtracting, multiplying and dividing wrongly.	Calculation error, miscalculation error, routine error, process skill, encoding, & value error	Durmuş (2019), Fahrudin and Pramudya (2019), Hutapea et al. (2015), Sari and Wutsqa (2019), Sekgoma (2023), Setiawan (2022), Supardi et al. (2021), Veloo et al. (2015), & Zulyanty and Mardia (2022)
5	Conclusion	Students are determining conclusions incorrectly. Students do not write conclusions.	Interpretation error & conclusion error	Durmuş (2019), Fahrudin and Pramudya (2019), Sekgoma (2023), & Veloo et al. (2015)

(1971) strategy were awarded 1 point, while students who did not complete this phase received a score of 0. The frequency and percentage of student responses at each level of Polya's (1971) strategy are then calculated. The items tested on the sample are, as follows:

1. A bookshop owner is using a ladder to hang a painting above the shelves, the ladder measures 17 m in length, and is inclined at 45° how far does the foot of the ladder extend from the wall to the ground?
2. A plane flying at 10,000 m is flying away from a person. The angle of elevation of the aircraft is 76° when initially observed. After 1 minute 15 seconds, the plane is at an elevation angle of 29°. Ignoring the person's height, what is the plane's speed in km/hr?
3. A ship is in the sea at 1 km from the top of a mountain. On top of the mountain, there is a lighthouse. The angle of elevation of the ship to the base of the lighthouse is 15°, and the angle of elevation to the top of the lighthouse is 20°. Calculate the lighthouse's height.

Data Analysis

The students' errors in solving the three test problems were analyzed based on the following categories: data error, concept error, strategy error, calculation error, and careless error (Fahrudin et al., 2019). Qualitative analyses were used to identify and explain students' errors in solving trigonometric problems based on the five general errors defined drawing on the literature review (see **Table 2**). After the students' data had been analyzed and divided into five different error categories, one student was randomly selected from each category. This selection aimed to explore the reasons for the errors and determine whether these errors correspond to errors described in the scientific literature. The students were given 30 minutes to answer the test items declared valid on a pilot sample.

RESULTS

Student Test Data

Table 3 shows the results of the sample participating in the problem-solving test. **Table 3** shows the initial analysis set (frequency and percentage) for each stage of Polya's (1971) problem-solving process, with the distribution of errors within the five specified categories.

Table 3. Frequency and percentage of students' responses (n = 30)

Variable	Data error	Concept error	Strategy error	Calculation error	Careless error
Understanding the problem	18 (60.0%)	12 (40.0%)	15 (50.0%)	13 (43.3%)	5 (16.7%)
Devising a solution plan	9 (30.0%)	16 (53.3%)	19 (63.3%)	15 (50.0%)	6 (20.0%)
Executing the devised plan	17 (56.6%)	20 (66.7%)	21 (70.0%)	18 (60.0%)	3 (10.0%)
Reviewing the solution	14 (46.7%)	16 (53.3%)	18 (60.0%)	16 (53.3%)	5 (16.7%)
Percentage (%)	48.3	53.3	60.8	51.7	15.9

Table 4. Percentage of students' errors in each task (number of the students/percentage)

Variable	Data error	Concept error	Strategy error	Calculation error	Careless error
Task 1	19 (63.3%)	10 (33.3%)	14 (46.7%)	12 (40.0%)	4 (13.3%)
Task 2	8 (26.7%)	15 (50.0%)	18 (60.0%)	14 (46.7%)	6 (20.0%)
Task 3	17 (56.7%)	20 (66.7%)	20 (66.7%)	17 (56.7%)	3 (10.0%)
Percentage (%)	48.9	50.0	57.8	47.8	14.4

The results show that errors are distributed significantly across the stages of solving mathematical problems, with the most significant percentage concentrated in the "understanding the problem" stage at 60% of data-related errors, indicating that students face difficulty in identifying and understanding the primary data of the problem. This difficulty dramatically affects their ability to make correct decisions in subsequent stages, which calls for improving data analysis skills and dealing with them effectively. In the "developing a solution plan" stage, conceptual and strategic errors increased, reaching 53% and 63%, respectively. This indicates significant challenges for students in understanding and applying basic mathematical concepts and choosing appropriate problem-solving strategies. Therefore, students need additional support in developing critical thinking and enhancing conceptual education that links theory with application.

During the "implementing the plan" stage, strategic errors were the highest at 70%, followed by calculation errors at 60%. These percentages indicate that students find it challenging to apply plans correctly and face problems accurately completing arithmetic operations. This situation requires improving arithmetic skills and enhancing practical training in solution strategies. In the "solution review" stage, the results showed that strategic and conceptual errors were still high, at 60% and 50%, respectively. This reflects that review alone is not enough to correct the basic errors that occurred in the previous stages, indicating the need to enhance the review process and encourage students to analyze their errors more effectively.

Although errors resulting from carelessness were the least frequent, they appeared at 15.9% in different stages, highlighting the problem of lack of focus and attention among some students while solving problems. Neglecting to review solutions or rushing to answer can lead to errors that could have been avoided. Therefore, it is essential to raise awareness of the importance of careful review and encourage students to focus and pay attention to details.

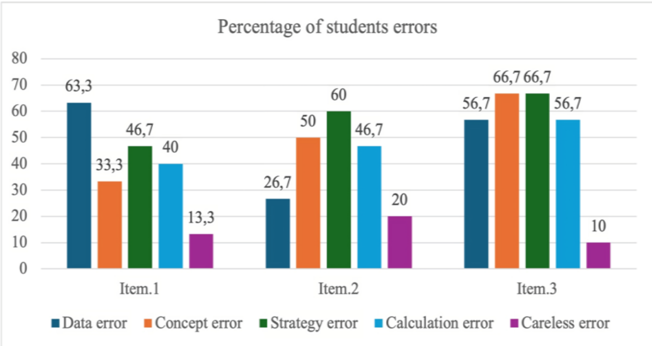

Figure 2. Percentage of students' errors for each test item analyzed based on Polya's (1971) strategy (Source: Authors' own elaboration)

In general, the percentages indicate that strategic, conceptual, and calculation errors are the most common, which requires enhancing students' planning and implementation skills and improving conceptual understanding and accuracy in arithmetic operations. This analysis emphasizes the need to adopt effective educational strategies that address these errors and develop students' performance in solving mathematical problems.

The students used Polya's (1971) strategy to solve all test problems. The students' errors were analyzed based on the five error categories: data error, concept error, strategy error, calculation error, and careless error. The student's results on the three tasks were, as shown in **Table 4**.

Figure 2 illustrates the proportion of errors students make in each of Polya's (1971) four steps. Students exhibited the highest percentage of data errors in the first item, whereas strategic errors were predominant in the second and third items.

A notable observation emerges from the analysis of the three items, indicating that errors predominantly occur during the verification of results. This happens when students must reassess their provided solution against the conditions outlined in the problem statement to determine its accuracy. Conversely, the initial step involving comprehension of the problem demonstrates fewer errors, suggesting a general understanding of the task requirements by students. However, challenges arise during the subsequent phases of problem-solving, particularly in the planning and execution stages, as students need help in devising and implementing strategies to address the given problem.

Analysis for Data Error

Table 4 indicates that students made the highest percentage of data errors while addressing Pythagorean theorem problems in the first item, with a rate of 63.3%. Conversely, the lowest percentage of data errors occurred in the second item, at a rate of 26.7%. Data errors arise when students inaccurately transcribe existing problem statements or incorrectly interpret given problems. This finding is substantiated by the results obtained from the interview with a student, as follows:

> Teacher: Is there any aspect of the question you find challenging?
>
> Student: Yes, sir, I encountered difficulty illustrating the ladder length on the diagram.
>
> Teacher: Could you explain the issue you faced?
>
> Student: The ladder length should be represented vertically, as I showed in the previous graph.
>
> Teacher: What about the 45-degree angle mentioned in the question? Where do you think it should be placed?
>
> Student: It should be adjacent to the right angle.

The dialogue indicates that some students needed to improve in illustrating the information presented in the question, particularly in accurately depicting the ladder. This is evident in their inclination to portray the ladder vertically (vertical height) rather than diagonally (lateral height). Furthermore, there needed to be a correction in depicting the angle formed by the ladder with the ground, as the student perceived this angle as adjacent to the right angle. Such errors arise when students encounter difficulties in translating and representing the data provided in the problem. Examples of student work errors due to data errors are shown in **Figure 3**.

Analysis for Concept Error

This error arises when students choose the wrong formula, theorem, or definition to solve a problem or when they need to include the necessary formulas, theorems, or definitions in their responses. The highest percentage of errors of this nature occurred in the third item, reaching a rate of 66.7%, whereas the fewest errors of this type were observed in the first item, with a rate of

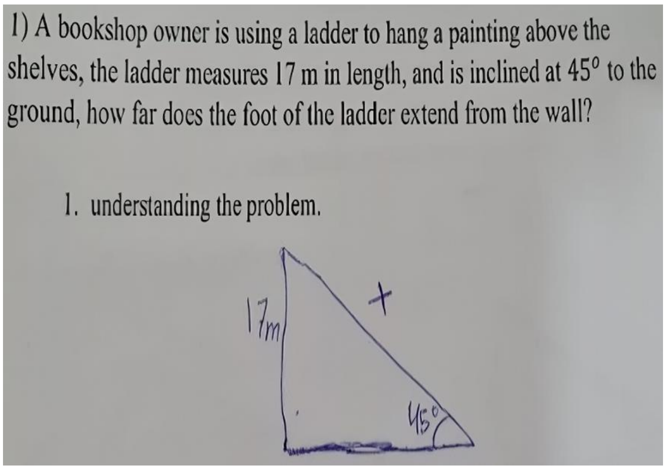

Figure 3. "Data" error (Source: Field study)

33.3%. The results obtained from the interview with a student validated the cause of this error, as follows:

> Teacher: How do you feel about the initial question? Do you believe there was an error at any point during the process?
>
> Student: Yes, sir, I believe I failed to select the appropriate trigonometric formula.
>
> Teacher: What specifically confused you about choosing the correct ratio?
>
> Student: I'm uncertain. I'm having difficulty distinguishing between the appropriate trigonometric ratios. I'm not sure.
>
> Teacher: You opted for the sine of the angle to determine the value of x. Do you think that choice was accurate?
>
> Student: Initially, I thought the trigonometric ratio I selected was correct, but now I realize I may need to correct a mistake.
>
> Student: I am confused between the trigonometric ratios sin and cos of the angle. I believe I made an error.
>
> Teacher: What is the accurate trigonometric ratio to solve the problem?
>
> Student: I understand that the sin of the angle equals the opposite over the hypotenuse, and cos of the angle equals the adjacent over the hypotenuse. I should have used the cos of the angle instead of the sin of the opposite.

Based on the analysis of the written tests and the interview conducted with the student, it is evident that the student made a compound error in their problem-solving approach. The primary mistake involved misrepresenting the data presented in the problem,

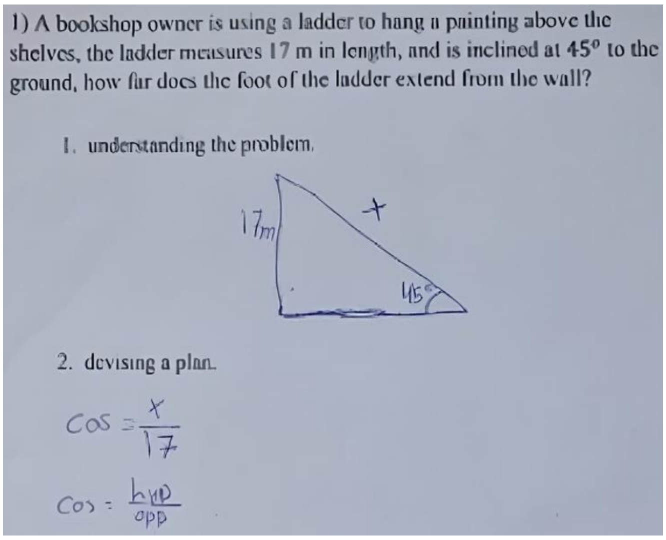

Figure 4. "Concept" error (Source: Field study)

while the secondary error was the selection of an incorrect trigonometric ratio. Specifically, the student erroneously asserted that the $cos = \frac{opposite}{hypotenuse}$, whereas the accurate relationship is $cos = \frac{adjusent}{hypotenuse}$. This particular type of error is exemplified in the student's responses to item test 1, as illustrated in Figure 4.

Analysis of Strategy Error

When addressing Pythagorean theorem problems, students may also make a strategic error as part of their approach. This occurs when students attempt to approach a problem at the correct level but employ unsuitable procedures or methods. On the other hand, students might operate at a suitable difficulty level but choose inappropriate data. The examination of students' data highlights a noteworthy prevalence of strategic errors in problem-solving tasks, with the highest percentage observed in the third item at a ratio of 66.7%. The results from the interview with a student affirm the cause of this error, as follows:

> Teacher: How do you assess the second task? Would you categorize it as easy or difficult?
>
> Student: I consider it easy.
>
> Teacher: Did you encounter any challenges at any point in the question?
>
> Student: I may have made an error in the final step of the solution, but I'm not sure.
>
> Teacher: Can you identify the specific mistake you think you made in that regard?
>
> Student: I'm not entirely sure, but I struggled to determine the value of x correctly.

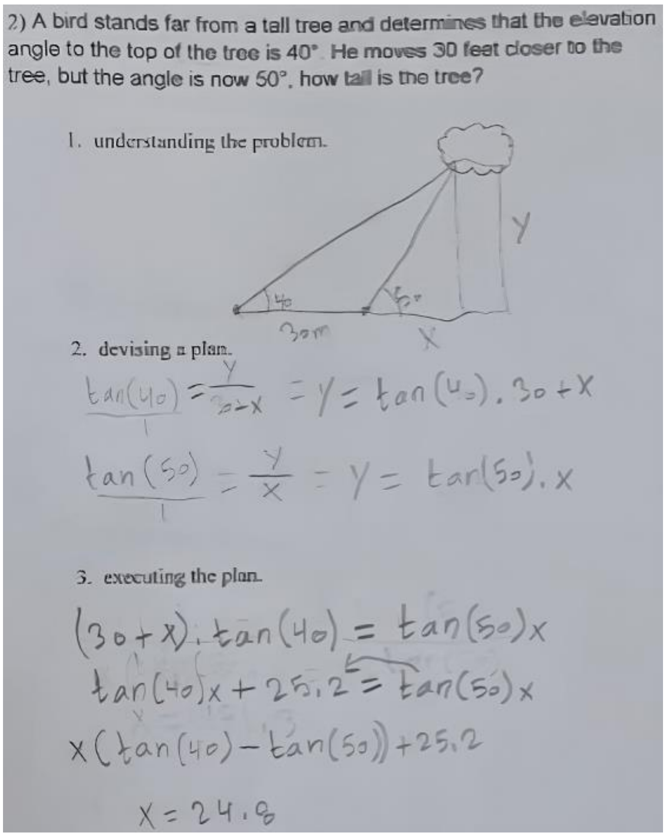

Figure 5. "Strategy" error (Source: Field study)

The results of the interview with the students showed that the stages and formulas for working on problems may be correct and compatible with the requirements of the questions, but students may make mistakes at some stage of the solution. Interviewing the student on the second item showed that the student can understand the question and represent it correctly. Still, he cannot complete the solution process by finding the value of x (by grouping similar terms and extracting a common factor). Examples of the work of students doing strategy errors can be shown in Figure 5.

Analysis for Calculation Error

The third item displayed the highest error rate within the calculation category at 56.7%, while the first item recorded the lowest percentage. Errors in this category occur when students inaccurately provide or record the symbols for mathematical operations or when they execute arithmetic operations, including incorrect addition, subtraction, multiplication, and division. To validate this case, an interview was conducted with a student, yielding the following confirmation:

> Teacher: Did you understand problem No. 3?
>
> Student: Yes, sir, I translated the data presented in the problem, and I took the required procedures for the solution.

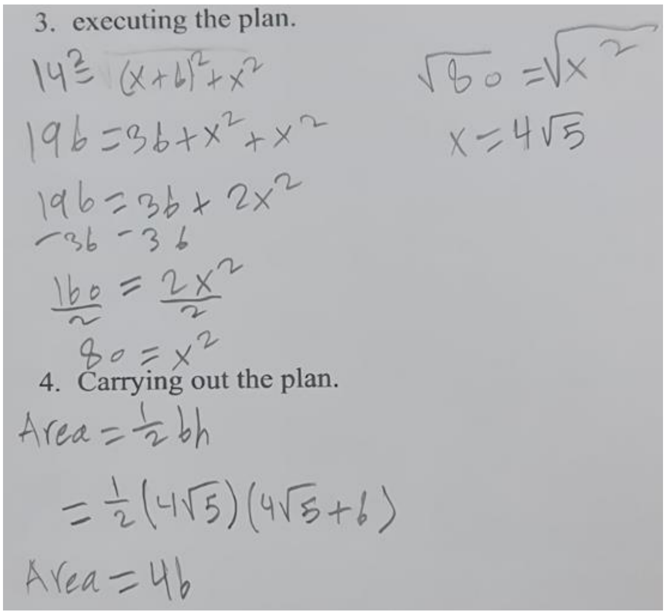

Figure 6. "Calculation" error (Source: Field study)

Teacher: Do you think you could have done better in calculating the area of a triangle?

Student: I think I should have calculated the roots.

Teacher: Why did you make a mistake?

Student: It seems that I needed to be more accurate and hastier in calculating the area.

After conducting an interview with the student and thoroughly reviewing the written test, it was identified that the student made a compound error in this problem. Initially, the student followed the solution steps but faced difficulty analyzing the perfect square $(x + 6)^2$. The second error involved a calculation error, as the student struggled with the accurate computation of roots. This occurs because, as mentioned in the interview, students tend to be less meticulous and hurried when solving problems. Figure 6 illustrates instances of students demonstrating strategy errors in their work.

Analysis for Careless Error

This error category arises when students need more accurate responses to questions or, in some cases, need to pay more attention to the questions. The percentage of occurrence for this error was relatively low compared to previous errors, recording 13.3%, 20%, and 14.4% for the three respective items. The outcomes of the interview with a student affirm one of the reasons for this error, as follows:

Teacher: Did you need help understanding question 3?

Student: No sir, I read the question and knew he wanted to find the area.

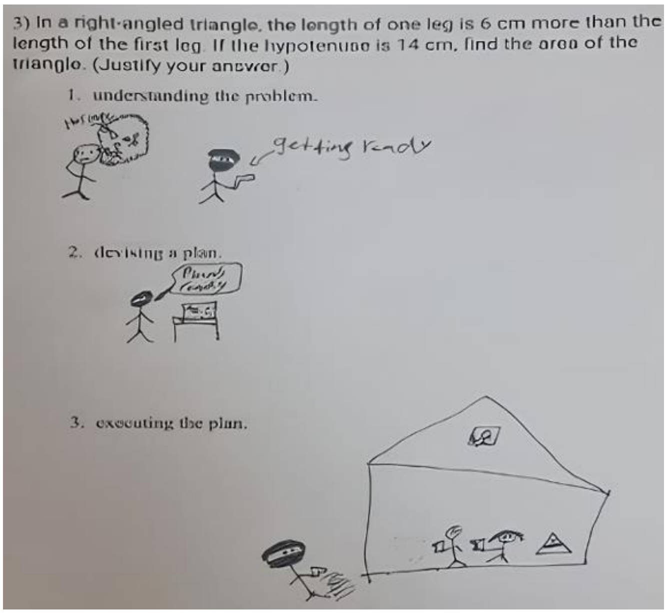

Figure 7. "Careless" error (Source: Field study)

Teacher: Did you try to represent the data given in the problem by chance?

Student: I don't know; I find it difficult to deal with this issue.

Student: This type is difficult for me.

Based on the findings from both the interview and the test paper, it is evident that the student struggles with problem-solving questions. The student needs to gain exposure to this problem type and employ Polya's (1971) problem-solving strategy to address such challenges. Examples of the work of students making strategy errors can be shown in Figure 7.

DISCUSSION

The results of the analysis of errors in the stages of solving problems according to Polya's (1971) steps showed that strategy errors were the most common at 60.8%, indicating great difficulty in developing and implementing solution plans. This was followed by conceptual errors at 53.3% and calculation errors at 51.7%, reflecting challenges in understanding concepts and performing operations accurately. Data understanding errors were also prominent at 48.3%, especially in the problem understanding stage, indicating the need to improve data analysis skills. Negligence errors were the least common at 15.9%, but they indicate the importance of focus and attention during the solution. These results highlight the need to enhance education directed toward deep understanding and effective strategies for solving mathematical problems.

Examining students' problem-solving errors based on the Pythagorean theorem has revealed significant patterns. As indicated by the analysis, the predominant errors are rooted in strategy application, with the highest percentage of mistakes falling within this category. A noteworthy observation is the students' struggle to employ algebraic operations correctly, underscoring a particular challenge in their mathematical approach.

A crucial aspect contributing to these errors is students' need for more grasp of the geometric meaning of the Pythagorean theorem. Additionally, their difficulties in determining the area of a right-angled triangle indicate a broader issue in understanding geometric concepts. This deficiency extends to challenges in applying trigonometry, suggesting a need for targeted support in this area.

Furthermore, the students needed help accurately modelling the data presented in the problems. The struggles in this aspect of problem-solving highlight a need for improvement in their analytical skills and the application of mathematical concepts to real-world scenarios. It was also apparent that some students encountered difficulty comprehending the problem statements, emphasizing the importance of refining their reading and interpretation skills.

These findings align with the outcomes of previous studies, consistently emphasizing the necessity for interventions addressing students' difficulties with mathematical representations, visualization, and the effective use of symbols. For instance, Fahrudin et al. (2019) conducted a study focusing on identifying errors made by students when tackling trigonometric equations. The findings revealed that strategic errors constituted the highest percentage of mistakes among students. Similarly, a study conducted by Veloo et al. (2015) observed that students made errors of negligence in specific tasks. Identifying these particular areas of challenge provides a solid foundation for educators to develop targeted strategies to enhance students' understanding and application of the Pythagorean theorem. By addressing these nuanced issues, educators can contribute to the holistic improvement of students' mathematical knowledge and problem-solving skills, fostering a more comprehensive and practical learning experience.

CONCLUSION

Analyzing students' problem-solving errors related to the Pythagorean theorem has revealed notable trends across various error categories. These identified errors include strategic errors, conceptual errors, data errors, arithmetic errors, and errors due to carelessness. Notably, each type of error displayed distinct patterns across different problem sets.

Data errors, which reflect difficulties interpreting information, showed consistent patterns across multiple problem sets. The first problem, in particular, exhibited the highest proportion of data errors, underscoring substantial challenges in understanding and applying data in context. Conceptual errors were predominantly observed in the third problem, indicating students' struggles with grasping geometric concepts associated with the Pythagorean theorem. This issue was especially pronounced in the third problem.

Strategic errors were also most prominent in the third problem, suggesting difficulties in employing effective problem-solving strategies, particularly within that context. Calculation errors were primarily identified in the third problem, highlighting challenges in performing accurate mathematical operations, specifically within that task. Carelessness errors were most evident in the second problem, suggesting issues related to attention to detail and precision in solution execution. Students faced significant difficulties in comprehending and modelling the problem in the second task, reflecting challenges in understanding the problem's context and accurately representing the data.

In terms of proposing solution plans, the third problem exhibited the highest frequency of errors, highlighting difficulties in formulating effective strategies for problem resolution. Errors in executing solution plans were most common in the first problem, indicating challenges in accurately implementing devised solutions. Lastly, the highest incidence of errors in verifying solution correctness occurred in the first problem, emphasizing difficulties in ensuring the accuracy of solutions in this task. This detailed analysis offers valuable insights into the specific areas where students struggle, providing a foundation for targeted instructional improvements and interventions.

However, the study faces several limitations that may affect the generalizability of its findings, the most significant being the small sample size of only 30 students, which limits the ability to apply the results to a broader population. Additionally, the study focused on specific tasks, which may restrict the applicability of the findings to other mathematical problems or topics. Time constraints and contextual factors, such as the timing of the tests and the learning environment, also influenced students' performance, potentially impacting the reliability of the results. Future research should aim to expand the sample size to include a larger, more diverse group of students to enhance the generalizability of the findings.

Despite these limitations, this research provides educators with valuable insights and a framework for developing tailored strategies to enhance students' understanding and application of the Pythagorean theorem. By addressing these specific challenges, educators can contribute to improving students' mathematical comprehension and problem-solving skills.

Author contributions: MAT: investigation, fieldwork, & first draft; **JD-P:** methodology, conceptualization, supervision, & review & editing; & **AM-S:** review & editing. All authors have agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Ethical statement: The authors state that this research is based on anonymous data from students at secondary education. Prior to conducting the study, authors obtained the authorization of the school and the families to perform the fieldwork and analyse the data collected. Informed consent was obtained from the parents or legal guardians of the participating students. The participating students were informed that their participation was voluntary and that they could withdraw from the study at any time without any negative impact on them. The authors further stated that all personal and sensitive data of the participants was treated with the utmost confidentiality, and the data was stored securely to ensure that it was only accessible by the authors. The data was anonymized so that the data used in the final analysis did not contain any information that would reveal the identity of the students, and all information was protected by high-security procedures.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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