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# Ethnomathematical connections between the production of coastal cheese, geometric solids, measurements, and proportionality: A study with a Colombian merchant

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Received 23 November 2024 - Accepted 21 January 2025

#### Abstract

The goal of this research was to explore the ethnomathematical connections between the elaboration of coastal cheese, geometric solids, units of measurement and proportionality. The theoretical basis is the ethnomathematical connections and types: internal, external and ethnomathematical meaning. The universal activities were considered: measuring, locating, counting, playing, designing and explaining; as well as the mathematical thoughts and quality referents issued by the Colombian Ministry of National Education. Basically, ethnomathematical connections are part of the extended theory of connections. The methodology was qualitative and developed in three stages: selection of the participant who was a cheese maker; data collection through a semi-structured interview for fifteen days to learn about cheese making, materials, techniques, procedures and observe their relationship with mathematics; data analysis aimed at exploring ethnomathematical connections. The results showed internal and meaningful ethnomathematical connections related to the use of conventional and non-conventional units of measurement (cap), geometric solids and round bodies (tank, "calambuco", and "zorrito"), proportionality (quantity of salt and number of calambucos) that were determined from the representation with units of capacity, cylinders, prisms, rules of three, among others. This research provides a substantial contribution to mathematics education, thus allowing the formation of mathematical concepts from the elaboration of coastal cheese and the rescue of language, universal cultural activities, artifacts and gastronomy.

**Keywords:** mathematical and ethnomathematical connections, mathematics education, daily practice, coastal cheese

# **INTRODUCTION**

Mathematical and ethnomathematical connections are a crucial topic in the field of mathematics education because they allow students and teachers to have a deeper and more lasting understanding of mathematical concepts, in fact, they can use them competently in different real-life situations (Berry & Nyman, 2003; García-García & Dolores-Flores, 2021; Rodríguez-Nieto et al., 2023b). Furthermore, connections are not only vital for mathematics but for social sciences, environmental sciences, engineering, arts, technologies, neurosciences, that is, they are applied in most of the procedures and processes of daily life that are not necessarily specific to mathematics (Campo-Meneses & García-García, 2024; Cantillo-Rudas et al., 2024; Rodríguez-Nieto et al., 2024).

In the broad research agenda in ethnomathematics and ethnomathematical connections, various studies are

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## **Contribution to the literature**

- This article provides a mathematical view of the treatment of milk and the production and marketing of coastal cheese in Manatí Atlántico, Colombia, considering the practical use of ethnomathematical connection tools that are part of the extended theory of connections.
- The knowledge of a merchant who, although he has not studied mathematics, uses it competently in his daily practice to obtain the best cheese for his clients is valued, that is, there is a human assessment.
- Priority is given to ethnomathematical connections, relating the knowledge used in daily practice with mathematical thoughts and basic learning rights (BLR and its acronyms in Spanish are DBA) suggested by the Colombian Ministry of National Education.

recognized that have delved into connections in the making of mud bricks and their implications in the classroom (Pabón-Navarro et al., 2022), connections in the making of the top of tapitas (Manchego et al., 2024), integration of ethnomathematics and ethno-modeling in the institutionalization of mathematical concepts taking into account the activities of fishermen in Indonesia (Sudirman et al., 2024). Also, the importance of the relationships between ethnomathematics and activities promoted with a STEAM approach to analyze diverse daily practices is observed (D'Ambrosio, 2020, Rodríguez-Nieto & Alsina, 2022; Rosa & Orey, 2021). In the context of geometry, multiple investigations have been carried out that implicitly report connections between artifacts and institutional mathematics with a view to a more graphically and visually competent education (Aroca, 2022; Salsabila et al., 2024; Sunzuma & Maharaj, 2022; Supriyadi et al., 2023).

Ethnomathematics has been heavily used to improve and create new teaching proposals considering the sociocultural aspects of students and teachers. Specifically, some studies in Indonesia and Thailand show that people's cultural knowledge enhances the teacher's knowledge in the research processes and he transmits it to his students for the understanding of arithmetic, geometric, statistical, etc. concepts (Payadnya et al., 2024; Putri et al., 2024; Sudirman et al., 2018, 2024; Sukestiyarno et al., 2023; Supriyadi et al., 2024). In Brazil, Rosa and Orey (2024) state that students learn mathematical concepts that are only present in the study programs, but it is important to recognize that the teaching of mathematics is inherent to the discourse of globalization stating that mathematical knowledge is universal. Particularly in Colombia, Mexico and other countries, ethnomathematics is considered relevant, assuming the construction of artifacts, indigenous textiles, among others (Acosta et al., 2024; Aroca, 2022; Cervantes-Barraza & Aroca, 2023; Mansilla et al., 2023; Rodríguez-Nieto et al., 2023a; Solano-Díaz et al., 2024).

In relation to gastronomy, two studies were identified that were interested in contributing to mathematics education from a cultural perspective, particularly with cheese. There are studies interested in valuing mathematics, but with different objectives (Pérez-Ortiz et al., 2024; Rodríguez-Nieto et al., 2022) as the main accompaniment to Mexican dishes. In this sense, De la Cruz and Buendía (2021) and Rodríguez-Nieto (2021) characterized the preparation of a Mexican tortilla to contribute to mathematics education. However, Rodríguez-Nieto and Escobar-Ramírez (2022) mention that

The literature on ethnomathematics shows that previous research has placed little emphasis on the connections between the art of gastronomy and the mathematics used to prepare a particular food product, but rather focuses on daily practices such as fishing, carpentry, masonry, livestock farming, or the production of artifacts such as kites, cabinets, masks, clothing, indigenous houses, the training of students and teachers in the context of ethnomathematics and geometry, among others (p. 971).

Therefore, the goal of this research is to explore the ethnomathematical connections between the elaboration of coastal cheese and geometric solids, measurements and proportionality to improve the understanding of geometric concepts from a sociocultural perspective and linked to the context of students, teachers and ordinary people.

# CONCEPTUAL FRAMEWORK

In this research, ethnomathematical connection refers to the relationship between the mathematical knowledge that people apply in their daily activities and the formal mathematics present in books and taught scientifically (Rodríguez-Nieto, 2021). These connections are divided into three types: internal, external, and ethnomathematical meaning (Font & Rodríguez-Nieto, 2024; Rodríguez-Nieto, 2020; Rodríguez-Nieto et al., 2023a). Internal connections deal with how a person relates different units of measurement (conventional or not) within the same system when making equivalences and conversions in their daily lives (Rodríguez-Nieto, 2020). External connections occur when a unit of measurement is used in a similar way in different measurement systems used in various daily practices (Rodríguez-Nieto, 2020). The ethnomathematical meaning connection refers to the meaning that a person attributes to a mathematical concept, linked to a cultural



Figure 1. Universal activities (adapted from Bishop, 1999)

object, artifact or measurement within their daily context (Rodríguez-Nieto, 2020).

Ethnomathematical connections favor the teaching and learning of mathematics from four points of view:

- They are relevant because they value mathematics in the daily practice carried out by a person where the researcher identifies a connection and relates it to institutionalized mathematics.
- They favor the understanding of mathematical concepts considering that the student solves mathematical problems based on real life and, in turn, suggestions on connections from curricular organizations are shared (Ministerio de Educación Nacional [MEN], 2006).
- They can not only be recognized in a single daily practice, but in several, from the same sociocultural context or from different towns, regions or countries, avoiding the local aspect of ethnomathematics when it is emphasized in a single daily practice (Rodríguez-Nieto & Escobar-Ramírez, 2022, p. 998-999).
- They contribute to the construction of ethnomathematical sequences and the design of tasks (Mansilla et al., 2023), which allows relating basic learning rights to mathematics to be identified in daily practices. In this case, the marketing of cheese links universal activities such as counting and measuring, found in the BLR proposed by the MEN (2006).

Ethnomathematical connections can also emerge from the activation of the universal activities proposed by Bishop (1999), but it is important to highlight that each community or person has their local activities with which they carry out their daily practice (**Figure 1**).

On the other hand, the Colombian Ministry of National Education has established 5 mathematical thoughts, which determine quality references for the generation of competencies in the classroom.

- 1. Numerical thinking and numerical systems, with the meaning of numbers and operations (MEN, 2006, p. 58).
- 2. Spatial thinking and geometric systems, with representations of shapes in space (MEN, 2006, p. 61).
- 3. Metric thinking and measurement systems, given magnitudes, units and measurement patterns (MEN, 2006, p. 63).
- 4. Random thinking and data systems, for stochastic decision making and representations (MEN, 2006, p. 64-65).
- 5. Variational thinking and algebraic and analytical systems, with algebraic and symbolic abstraction, to deal with variations with unknowns (MEN, 2006, p. 66).

## **METHODOLOGY**

This research is qualitative and framed in a case study (Stake, 1995), due to its descriptive nature in the life experience of a merchant who shows mathematical notions immersed in daily practices. Thus, due to the theoretical support based on ethnomathematics, the research is based on an ethnographic approach (Aroca, 2018b; Cohen et al., 2018; Restrepo, 2016; Fuentes, 2019) developed in three stages:



Figure 2. Data analysis according to the proposed theoretical constructions (Source: Authors' own elaboration)

- selection of the participant, who is a cheesemaker by trade and one of the most recognized in the town;
- (2) request and application of a semi-structured interview, and
- (3) analysis of the data and contrast with the theoretical foundation and mathematical thoughts suggested by the MEN (2006).

#### **Participant and Context**

The voluntary participant in this research was Mr. Ángel Olivero, who is 52 years old, has an academic high school education and has more 40 years of expertise in agriculture, livestock, milk treatment and production of coastal cheese. Currently Mr. Ángel has about 6 years with his own and official business where he manufactures his product day by day, which positions him as a cheesemaker by trade in Manatí. Manatí is a riverside municipality located in the south of the Atlántico department in Colombia, whose main economic activity is cattle raising, agriculture and commerce. In fact, these enterprises, such as the cheese factories, are a source resulting from daily work, in this case, the treatment and handling of the milk that farmers obtain on their farms from milking cows.

## **Data Collection**

The data were collected, based on the significance of mathematical ethnography raised by Londoño-Agudelo et al. (2024) from the reflections on mathematics education from ethnomathematics (Aroca, 2018a) as an opportunity to learn from the strange forms of mathematical representation of people, preserving the authenticity and autochthony of the daily practices of the cultural context (Bishop, 1999; D'Ambrosio, 2014; Londoño-Agudelo et al., 2024). Regarding the search for information with qualitative research, the semistructured interview technique was implemented (Hernández et al., 2014), with 15 days of accompaniment and participant observation. This approach seeks immersion in the context, to detect possible signs of personal mathematical activity in the manufacture of coastal cheese (D'Ambrosio, 2014).

#### Data Analysis

The analysis of the information was based on the provisions of Hernández et al. (2014) with the qualitative analysis of detailed information by social phenomena or highly recurring topics, having as support, the semistructured interview and the theoretical foundation with mathematical thoughts (MEN, 2006), universal activities (Bishop, 1999) and types of ethnomathematical connections (Rodríguez-Nieto, 2020). Thus,

- (1) the semi-structured interview was transcribed in detail,
- (2) readings were made that allowed the identification of mathematical clues and codes,
- (3) ethnomathematical connections between daily practice and institutionalized mathematics were identified, and
- (4) a cluster of ethnomathematical connections was reported and established with considered aspects of the practice, the resulting implicit mathematical objects, the categorization by mathematical thoughts (numeric, metric, spatial, variational) and universal activities.

For example, the cheesemaker claims that he uses a standard or unit of measurement that he has tested throughout his work, which contains 10 cc and serves to measure the rennet and its perfect proportion in the milk (tapa). Thus, an internal ethnomathematical connection is shown with evidence of metric thinking (universal activity: measuring) in the units of capacity and conversion of units (see **Figure 2**).

It should be noted that these equivalences vary according to the quantity of milk to be cut, however, the pattern and/or unit of measurement established by the cheesemaker from his expertise is highlighted, as well as how conventional units of measurement have been adapted to non-conventional ones from equivalent reference objects.



**Figure 3.** The "zorrito" and ethnomathematical connection between "zorrito" and rectangular prism (Source: Authors' own elaboration)

#### **FINDINGS**

For the sequential purposes of this research, the results constitute the procedural stages for the treatment of milk and the production of coastal cheese (gastronomy), thus revealing all the ethnomathematical connections present in this daily practice, associated with the types of connections, universal activities and mathematical thoughts.

#### **Procedural Steps For Milk Treatment**

#### Stage 1. Milk collection and treatment

For this first moment, the cheesemaker claims that he starts his day at 6:00 a.m. at his respective workplace (cheese factory), where he proceeds to receive all the milk that arrives (see excerpt from the interview).

I: Good morning, Mr. Angel, nice to see you. Today we would like to respectfully interview you about your work and expertise in making coastal cheese.

P1: Good morning, my son, yes of course.

I: Tell me, Mr. Angel, how do you make cheese?

P1: Well, I make cheese every day, from 6:00 a.m. until 12:00 p.m. when I finish. What I do is that I receive the people in my cheese factory who are in charge of bringing the milk, and I also send workers with a "zorrito" and a motorcycle to the plots for the milk.

The "*zorritos*" represent the means of transport used to move the milk from the corrals or farms to the cheese factory (see **Figure 3a** in **Figure 3**). These artifacts are made of iron and are joined together with welding and some base boards to form a structure with two wheels that is pulled by motorcycles, providing comfort in transport. From this artifact, the first ethnomathematical connection of our results (see **Figure 3b** in **Figure 3**).

From the connection between mathematics linked to the cultural and institutional component, the structure that recreates the "zorrito" is fully associated with spatial thinking and geometric systems, with a



**Figure 4.** The "zorrito" and ethnomathematical connections with dimensions of the rectangular prism and its elements (Source: Authors' own elaboration)



**Figure 5.** Ethnomathematical connections between the "zorrito's" tires and the concepts of circumferences: elements, circle and cylinder (Source: Authors' own elaboration)

rectangular prism of dimensions length (b) = 104 cm, width (i) = 80 cm, height (h) = 41 cm. From this first connection you can see the edges (12 edges), vertex (8 vertex or joining points) and faces of the solid (6 faces or fronts) in perfect harmony, thanks to the metal angles that shape the structure (see **Figure 4**).

In the "zorrito" the circular tires with rim 13 are identified, the front of which outlines a circumference and the entire rim shows a cylinder, while the center of the same where the "balineras" are located, represents the center of the circumference through which the train passes and joins the radius of the same (see **Figure 5**). As the day goes by, the milkmen and workers arrive with their "zorritos" loaded with milk, so it is necessary to package it and make some sales (see excerpt from the interview).

I: How do you pack the milk?

P1: Well, there are people who bring it in small plastic "pimpinas" (containers), calambucos (containers) and so on. When they arrive, I have to measure them. For people who bring a few liters of milk to sell, I measure them with this container that makes one liter and it's faster. I also use this container in case someone comes to buy milk for breakfast or something like that.

The utensils that people bring to the cheese factory are "pimpinas" or pasta buckets, which represent a type



**Figure 6.** The "pimpina" and ethnomathematical connection between the "pimpina" and rectangular prism (Source: Authors' own elaboration)



**Figure 7.** Volume of a "pimpina" and conversion of units to rectify the capacity of 20 liters of milk (Source: Authors' own elaboration)

of closed object with a circular mouth on the top that has a cap (see **Figure 6a** in **Figure 6**).

From here, we obtain the second ethnomathematical connection, from metric and geometric thinking, since the "pimpinas" have a capacity of 20 liters and their structure is that of a rectangular prism with a height greater than the width and length and bordered at each corner (see **Figure 6b** in **Figure 6**).

In fact, the dimensions of this pasta (plastic) artifact correspond to length (l) = 25.5 cm, width (a) = 20.9 cm, height (h) = 37.5 cm and the study that can be carried out from them, concerns the rectification of the volume in units of cm<sup>3</sup> (see **Figure 7a** in **Figure 7**) to be converted to liters (conversion of units with capacity magnitudes) and verify the mathematical correspondence between the data given by the participant (cultural and informal practice) and the mathematical procedure (institutional and formal practice) which would be 20 liters. It should be noted that all this is developed from metric thinking (see **Figure 7b** in **Figure 7**).

The workers who transport the milk in the "zorritos" bring with them calambucos, which are utensils to contain the milk (see excerpt from the interview):

I: Excellent! What is calambuco and how do you measure that milk?



**Figure 8.** The "calambuco" and ethnomathematical connections between "calambuco", truncated cone and cylinder (Source: Authors' own elaboration)



**Figure 9.** The liter pot and ethnomathematical connections with conventional units of capacity and units of measurement (Source: Authors' own elaboration)

P1: Well, I work with calambucos, which are these aluminum-like containers that hold 40 liters each. There are several types of calambucos, I only work with the 40-liter ones and this liter container in case you need to measure a little milk more quickly.

As the participant claims, the calambucos contain a maximum amount of 40 liters of milk inside and they are used to collect the milk from the farms or plots to the cheese factory (see **Figure 8a** in **Figure 8**). In this sense, the third ethnomathematical meaning connection arises that is linked to spatial thinking and geometric systems, since it can be seen that the calambuco constitutes a geometric body composed of 2 cylinders and a truncated cone. These considerations arise from the elongated shapes of the calambuco, the variations in its shape and its circular bases (see **Figure 8b** in **Figure 8**).

Having concluded this, another artifact that the participant mentions is the liter pot or container, which consists of a pasta container with markings that suggest exact measurements of milk. From here, we obtain the fourth ethnomathematical connection within metric thinking, since the container measurements show 1 liter divided into 4 parts, which refers to conventional measurement units. As mentioned above, the participant uses this pasta utensil to sell some milk to neighbors, which increasingly shows that the measuring rule for buying and selling milk is the liter (see Figure 9).



**Figure 10.** Ethnomathematical connections between the "varilla" for measuring milk and scale with natural numbers and units of capacity (Source: Authors' own elaboration)

#### **Procedural Steps For the Production of Coastal Cheese**

#### Procedural step 1. Milk measurement and ideal weight

In this second stage, the cheesemaker claims that it is necessary to measure the milk, so a new device responsible for this magnitude is included: the "varilla" (see excerpt from the interview).

I: How do you measure milk?

P1: Well, I pour the milk into the calambucos and put the "varilla" in it.

I: What is "varilla"?

The measuring "varilla", for its part, represents a measuring device in liters (see interview excerpt of ethnomathematical meaning connection). In fact, this is where our fifth ethnomathematical connection arises, associated with numerical and metric thoughts, since the "varilla" has the shape of a scale from 0 to 40 with natural numerical values (positive numbers), where each line represents the location of a number and likewise, the distance between values and interval of natural numerical value (segment), determines the measurement in liters of the quantity contained in the 40liter calambucos (see Figure 10).

P1: Well, the dipstick is this one, which is like a metal ruler that has a number from 1 to 40. I put the dipstick into the container and up to where the quantity of milk is indicated with the dipstick, that is what the dipstick measures. If it marks at 10, it is 10 liters, at 20 it is 20 liters and so on. If the container is already filled up to the 40 marks, then you know there are 40 liters.

After measuring the milk in the calambuco with the "varilla", the cheesemaker proceeds to verify the quality of the milk to check if it is pure or adulterated with water, that is, the ideal weight is verified (see excerpt from the interview).

I: Ok Mr. Angel, tell me, after this, what do we do?



**Figure 11.** Ethnomathematical connection between the refractometer and scale of values with natural numbers (Source: Authors' own elaboration)

P1: Well, after this, I take this device to measure the weight of the milk.

I: How so Mr. Angel?

P1: Well, son, I'll tell you that there are malicious people who come and pour water into the milk to get paid more money. I don't trust anyone, so I came and take a drop of milk and put it in this refractometer.

In this sense, the refractometer validates the exact weight that the milk must have in order to be accepted at the cheese factory (see interview extract). In fact, this is where the sixth ethnomathematical connection arises, linked to numerical and metric thinking, since the way the refractometer is displayed shows a scale of values as a numerical line (see **Figure 11**).

I: Ok Mr. Angel, how do you know if the milk is good or bad?

P1: Well, my son, this shows you a scale with several numbers from 0 to 40 and it is all blue. The ideal weight of milk is nine and above, this means that if there is a blue band from nine upwards, it is good. If on the contrary, the band is eight or less from eight upwards, it means that the milk has water and I have to throw it away.

#### Procedural step 2. Packaging the milk in larger tanks

After measuring the milk in the barrels and verifying the ideal weight, the cheesemaker states that it is necessary to repackage the milk in larger containers for the following stages (see excerpt from the interview).

I: Oh, ok, Mr. Angel, interesting. What do you do next?

P1: After measuring the milk and seeing if it's good, I bottle it in these large tanks that each hold 200 liters. What I do is fill the tanks to the top so I can cut the milk and every time I pour it in, I strain it so that the dirty stuff doesn't go in.



**Figure 12.** The 200 L tank and its ethnomathematical connection with the cylinder (Source: Authors' own elaboration)



**Figure 13.** 200 L tank and its ethnomathematical connection with cylinder volume, unit conversion and rectification of maximum capacity of 200 L (Source: Authors' own elaboration)

Thus, the milk is transported to the 200-liter tanks and previously strained (see **Figure 12a** in **Figure 12**), resulting in our seventh ethnomathematical connection associated with geometric, metric and numerical thoughts, since these tanks have a cylindrical shape with specific dimensions: height (h) = 81.2 cm and a diameter (d) = 56 cm (see **Figure 12b** in **Figure 12**).

Based on the units of measurement and the capacity these tanks in relation to their content, the of cheesemaker states that "each one holds 200 liters"; that is, each tank is capable of holding a maximum of 200 liters of milk (mathematics in daily practice). In this sense, this statement can be verified analytically and procedurally, remembering that the quantity of milk in the tank is associated with the volume of the cylinder in  $cm^3$ . Thus, the dimensions of the cylinder h = 81.2 cm and d = 56 cm will be considered, remembering that to find the volume we need the radius of the circular base and it is associated with d, so if d = 56 cm, then r is half of  $d_r r = 28$  cm (see Figure 13a in Figure 13). It should be noted that we will obtain the volume of the tank in  $cm^3$ as a standard unit, so to verify that the tank capacity is 200 L, specific conversions of the capacity magnitude from  $cm^3$  to liters will need to be performed (see Figure 13b in Figure 13).

Along these lines, the participant states that each large 200 L tank can contain as many calambucos with milk as it requires (see interview excerpt):



**Figure 14.** Relationship between tank and "calambuco" and their ethnomathematical connections with the simple direct rule of three (Source: Authors' own elaboration)

I: How many calambucos of milk are put in that tank?

P1: This tank is 200 L, so about 5 full calambucos of milk are put in.

The cheesemaker is stating that there is a magnitude relationship between the 200 L tank and the number of calambucos (see **Figure 14a** in **Figure 14**). Right here, the eighth ethnomathematical connection occurs, which is associated with numerical thinking and more specifically with the simple rule of three from direct proportionality, since there is a joint relationship between two magnitudes in a direct way, so the greater the number of 200 L tanks, the greater the number of calambucos with milk that are required to fill them (see **Figure 14b** in **Figure 14**).

#### Procedural step 3. Cutting off the milk

When bottling the milk in the large 200 L tanks, the cheesemaker says that it is necessary to curdle the milk (see excerpt from the interview):

I: What is this about curdling milk, Mr. Angel? Do you use a knife or something?

P1: (...), no, my boy. What I do is add rennet to make it hard and turn into a hard gelatin.

For the purposes of this study, rennet is a dark liquid chemical that comes in a blue container and makes milk hard (see **Figure 15a** in **Figure 15**), in this case, there are some metric restrictions for the use of rennet and is that the amount to be used is proportional to the amount of milk to be curdled (see interview extract).

I: What is rennet?

P1: It's this chemical, it's called titanium rennet. What I do is pour 2 caps into the tank and stir for 3 minutes with this spatula so that all the milk receives rennet and spreads.



**Figure 15.** Rennet and its ethnomathematical connections in unit conversion (Source: Authors' own elaboration)



**Figure 16.** Ethnomathematical connections between the use of rennet with cap and the simple direct rule of three (Source: Authors' own elaboration)

I: How do you know that you have to pour 2 caps? Do you have any measurements?

P1: Well, my son, it says that it's 1 ml of those syringes for every 10 liters of milk. From the time that I've been doing this, I know that a cap of rennet is put in 10 ml of syringe, which would be for 100 liters, right? So, for 200 liters I come and pour the two caps and it's easier.

In this case, it can be observed that the participant ratifies the use of a non-conventional unit of measurement as a reference in the use of rennet. In this way, the ninth ethnomathematical connection appears, which is closely associated with metric thinking with the implementation of conventional units (1 ml of syringe) and non-conventional units, with the appearance of the rennet cap that is equivalent to 10 ml of syringe, making conversions between these two metric reference systems (see **Figure 15b** in **Figure 15**).

Likewise, the use of numerical thinking can be seen explicitly in the established proportions (2 caps of rennet for every 200 liters), implementing simple direct rules of three that relate the two magnitudes mentioned above (see **Figure 16**).



**Figure 17.** The "canalete" and ethnomathematical connections between "canalete" and geometric bodies (Source: Authors' own elaboration)

#### Procedural step 4. Split the dough and make the brine

After having cut the milk, stirring it with the "canalete", comes a crucial stage and that is to break up the dough or gelatin that coagulates with the effect of the rennet to proceed with the other stages (see excerpt from the interview):

I: Ah, ok, Mr. Angel, the part about the rennet and the curdling of the milk is very interesting. After this, what do we do?

P1: Okay, son, we leave it alone to rest for about an hour. After 1 hour we come and break up the dough, which is the milk that becomes hard like gelatin. When the hour has passed, the milk becomes hard and we break it up or crack it gently with this spatula.

After an hour and breaking the dough, the device called a "canalete" is used (see **Figure 17a** in **Figure 17**), which is used in other practices such as fishing and is made of wood with one elongated part and another with a closed curve. In fact, here arises our tenth ethnomathematical connection which is closely linked to spatial thinking and geometric systems since the elongated base of the "canalete" by which it is held, has the shape of a parallelepiped with rectangular faces, while the other part, which is the one that breaks the dough when making up-and-down movements with the elongated base, has an elliptical face and is elongated in shape, it seems to be a type of somewhat flat semi-ellipsoid (see **Figure 17b** in **Figure 17**).

After breaking up the dough, it is left to rest (see interview excerpt) in order to proceed with the separation of the mixtures.

I: After breaking it, what do you do?

P1: Well, my son, what I do is leave it for a while, that dough collects itself at the bottom and settles. There is a greenish "suero" on top, what I do is take out all the sweet "suero" and leave the dry dough below.



**Figure 18.** Ethnomathematical connections between quantity of sweet "suero" and simple direct rule of three (Source: Authors' own elaboration)

I: Ok, Mr. Angel, what do you do with that sweet "suero"? And how much sweet "suero" do you get out of there, more or less?

P1: Well, for sure, we use that sweet "suero" to make cooked butter, which is another separate activity, and we take the rest to feed the pigs, dogs, and other animals that are raised in the yards of the houses. From those 200-liter tanks, I get about 2 40-liter calambucos and 20 liters separately.

Sweet "suero" is the remaining material that results from solidifying milk by cutting it with rennet and turning it into dough. This liquid has various uses and according to the experience of the cheesemaker, there is a specific amount that results from cutting a certain amount of milk (see Figure 18a in Figure 18). In fact, we have discovered our eleventh ethnomathematical connection associated with numerical thinking, since if for every 200 liters there are 2 "calambucos" of 40 L + 20 L apart, it is intuited that 2(40 L) + 20 L = 80 L + 20 L =100 L, so we affirm that 200 L of milk when cut gives us 100 L of sweet "suero". Here, we observe a simple direct rule of three relating magnitudes proportionally, resulting in that, the greater the amount of milk cut, the greater the amount of "suero" (see Figure 18b in Figure 18).

After separating the dough to dry it, the brine is made, which consists of adding salt to the dough to proceed with packaging (see excerpt from the interview):

I: So, Mr. Angel, nothing is wasted, everything is used here. Tell me, after the dough is ready, what do you do?

P1: Well, son, when I have the dough alone, I come and add salt to make the "salmuera" or salty dough.

I: Tell me, Mr. Angel, how much salt do you add? How do you measure it?



**Figure 19.** Ethnomathematical connections between measured salt use and quantity of soured milk (Source: Authors' own elaboration)

P1: There are people who add it by eye and try it, but I don't like it because it can be plain or salty. When I started, I would try with the salt weighed in this weight until I got it right, you know. I come and add 10 pounds of salt to 200 liters, and so if you want for 100 liters it would be 5 pounds and so more or less you add your share of salt.

The amount of salt added to prepare the salmuera is proportional to the amount of milk that is curdled, and it is weighed with a needle scale (see **Figure 19a** in **Figure 19**). Therefore, it is the twelfth ethnomathematical connection related to numerical thinking and metric thinking, since concepts related to conventional units of measurement of dough (pound) and capacity (liter) are addressed, when the cheesemaker claims that there are 10 pounds of salt for every 200 L of curdled milk (dough). Likewise, the concept of the direct rule of three is observed, specifying a proportionality between the amount of curdled milk and the amount of salt, so that, the greater the amount of curdled milk, the greater the amount of salt for the dough (see **Figure 19b** in **Figure 19**).

# Procedural step 5. Pack the brine into bags and molds to form the cheeses

When making the salmuera, one of the most crucial stages of this procedure comes and that is the packaging of the dough with salt to achieve the cheese mold (see excerpt from the interview):

I: Ok Mr. Angel, I think it's good that you've learned so much in your work. After adding the salt, what do we do?

P1: We stir with the spatula so that everything comes together and the salmuera is ready. Now that we have the dough ready, we're going to pack it to make the cheese.

I: Ok, now comes the good part, explain to me how is that dough packaged to make the cheeses?



**Figure 20.** The punch bowl and its ethnomathematical connections with the inverted truncated cone (Source: Authors' own elaboration)



**Figure 21.** Ethnomathematical connections between the basket and the rectangular prism (Source: Authors' own elaboration)

P1: Get what we do. First, we place these 2 punchbowls and on top of them we put one on top of the other these baskets that I have to shape the cheeses, there would be 4 baskets, two for each punch bowl. In the top basket I come and put this white cloth cover that a seamstress makes, it looks like a pillowcase, but it's made of white cloth, so that the dough falls here and doesn't spill out but rather is well drained.

I: Excellent, I see that this cover fits precisely in the mouth of the basket.

P1: Yes, the lady who makes them makes sure they fit snugly.

The packaging process begins with the punch bowl (see **Figure 20a** in **Figure 20**), which is a clay artifact used to make containers and is mostly used in village homes to wash clothes. In this sense, our thirteenth ethnomathematical meaning connection appears, which is linked to spatial thinking and geometric systems, since the punch bowl is shaped like an inverted truncated cone; that is, a sectioned cone (see **Figure 20b** in **Figure 20**).

Likewise, the baskets that are located in the punch bowls (one as a base for the other, and the next as a container for the cheese with the cover), are pasta artifacts with faces having holes through which the "suero" is evacuated and the top is clear to throw out the content of the bowl (see Figure 21a in Figure 21).

In this sense, the baskets give rise to the appearance of the fourteenth ethnomathematical connection, which



**Figure 22.** The white cloth and its ethnomathematical connections with the rectangular faces and surface area of the basket (Source: Authors' own elaboration)

is linked to spatial thinking and geometric systems and metric thinking, since these baskets are shaped like rectangular prisms, whose parallel faces are congruent rectangles and have dimensions of: length (b) = 38 cm, width (l) = 28 cm and height (h) = 23 cm (see **Figure 21b** in **Figure 21**).

After placing the baskets, the cover is placed, which is a kind of white cloth blanket as a pillowcase and has the ability to be filled with brine to contain the mass and make the salted whey come out (see **Figure 22a** in **Figure 22**). In fact, this is where the fifteenth ethnomathematical connection of meaning associated with spatial thinking and geometric systems arises, since the two joined sides of the roof have a rectangular shape that adapt to the interior of the basket and cover the four lateral faces (*Rl*) and a basal face (*Rb*), respectively.

To know the area of the white cloth, it is enough to calculate the surface area of the layette, not including the upper basal face. The rectangular bases of the prism are congruent evenly, therefore, if R1 = R2 we will say that both share the same base and height: b = 28 cm and h = 23 cm. R3 = R4 with b = 38 cm and h = 23 cm, and R5 with base: b = 38 cm and h = 28 cm. It is known that the area of a rectangle is the product of the base times the height, and to find the surface area we proceed to add the area of the five faces described above, thus obtaining a surface area of 4,100 cm<sup>2</sup> (see Figure 22b in Figure 22).

After preparing the punch bowl with the baskets and the cover, the dough is poured in with specific quantities (see excerpt from the interview):

#### I: Then what do we do?

Well look, from this 200-liter brine I take out 6 buckets of dough with this yellow bucket that holds about 25 to 30 liters and I distribute them in two baskets for two cheeses, each bucket of dough gives about 10 pounds of cheese, I pour the 3 buckets into each bag in the baskets and begin to squeeze little by little until it takes the shape of the basket and is dry and hard.



**Figure 23.** The yellow bucket and its ethnomathematical connections with the solid cylinder (Source: Authors' own elaboration)



**Figure 24.** Ethnomathematical connection between yellow bucket, volume and unit conversion (Source: Authors' own elaboration)

The yellow bucket is a reused pasta utensil that originally held paint, but after being washed, it is used to hold water and other mixtures (see **Figure 23a** in **Figure 23**). Thus, it is the sixteenth ethnomathematical connection associated with spatial thoughts and geometric systems and metric thinking, since the yellow bucket has a cylindrical structure with dimensions of height (h) = 38.5 cm and diameter (d) = 30 cm, as reported by Rodríguez-Nieto and Escobar-Ramírez (2022) in their ethnomathematical study of Guandú soup (sancocho) in Sibarco, naming it as a gallon according to the cooks participating in their study (see **Figure 23b** in **Figure 23**).

It should be noted that, according to the cheesemaker, the yellow bucket can contain from 25L to 30L of water or milk, which result in 10 pounds of cheese's dough, respectively. This process allows us to verify the exact capacity of the bucket from the volume of the cylinder, recognizing by its dimensions that if d = 30 cm, then the radius is given by r = 15 cm and thus the formula can be applied to find the volume of the cylinder (see **Figure 24**).

On the other hand, metric thinking becomes evident, since there are conversions of capacity units. The cheesemaker states that the resulting brine of 200 L is equivalent to 6 yellow buckets and that each yellow bucket gives 10 pounds of cheese, therefore: if 200 L = 6 yellow buckets of dough =  $6 \times 10$  pounds = 60 pounds. From a 200 L tank, 60 pounds of cheese result. Now, it



**Figure 25.** Ethnomathematical connection between buckets of dough per tank, pounds of cheese per tank and simple direct rule of three with proportionality (Source: Authors' own elaboration)

also becomes clear that 3 yellow buckets of dough are poured into each bag in the baskets, so from a 200 L tank, 60 pounds of cheese are produced, distributed in two 30 pounds cheeses, respectively (see **Figure 25**).

After packing the dough into the bags with baskets and squeezing it until it is dry, the respective baskets with the cheese are placed on the counter and the bag is collected, leaving the salted "suero" in the punch bowl (see excerpt from the interview).

I: Mr. Angel, what is that "suero" left in the punch bowl for?

P1: Do you remember the sweet greenish "suero" I told you about? Well, they mix this one with this one that has salt and make butter, although there are people who use it to prepare spicy food.

#### Procedural stage 6. Cheese formation

After collecting the bag and placing the cheese on the counter, some weight is added to the mold so that the cheese finishes compacting (see interview excerpt).

I: Great! Tell me after the dough is squeezed and the basket is compacted by the mold, what do we do?

P1: I wrap the remaining bag, and I carry the basket to raise it to this counter where it will continue to be squeezed.

I: What or who continues to squeeze it?

P1: Icome and set up this board that is almost the size of the mouth of the basket, and finally I put this press on it so that it is squeezed little by little by the weight.

The board mentioned by the cheesemaker is a piece of wood that fits over the mouth of the basket, in this case, the top of the basket (see **Figure 26a** in **Figure 26**). On the other hand, the press is made up of a piece of pipe



**Figure 26.** The wooden board and its ethnomathematical connection with the parallelepiped (Source: Authors' own elaboration)



**Figure 27.** The press and its ethnomathematical connection with the cylinder (Source: Authors' own elaboration)

# filled with concrete and is placed on top of the board (see **Figure 27a** in **Figure 27**) (see interview extract).

I: Who made that board and press?

P1: I made that board and polished it to make it smooth. For the press I took a long round piece of pipe, filled it with concrete and put that rod on top to hold it. This press is good, it finishes squeezing the cheese.

Thus, the seventeenth ethnomathematical connection associated with spatial thinking and geometric systems arises, since the parallelepiped mathematical object with rectangular faces whose height is greater than the base (wooden board) (see **Figure 26b** in **Figure 26**) and the cylinder (press) (see **Figure 27b** in **Figure 27**) becomes evident.

Finally, the dough is left to wait a certain amount of time, then it is turned over so that it is pressed on the other side and compacted evenly (see excerpt from the interview):

I: Ok Mr. Angel, what do we do next?

P1: We leave it for 20 minutes to press it with the press, then we turn it over so that the other side is also pressed and we leave it for 20 more minutes.



**Figure 28.** The block of cheese and its ethnomathematical connection with the rectangular prism (Source: Authors' own elaboration)

You see how it is going; it is taking the shape of the basket and it is sealed.

After 40 minutes of pressing on both sides, the cheese is removed from the casing, packed in a plastic bag, returned to the basket and refrigerated (see excerpt from the interview).

I: Excellent! After this I imagine it can be stored.

P1: Sure, I come and take it out of the basket, take it out of the cover and put it in a black bag, then I put it back in the basket and arrange it well in this freezer so that it hardens more.

The resulting compacted cheese appears in its final form as a hard dough (see **Figure 28a** in **Figure 28**). In fact, this is the eighteenth and final ethnomathematical connection from this study, linked to spatial thinking and geometric systems, since the cheese has the shape of a rectangular prism congruent to that of the basket (see **Figure 28b** in **Figure 28**).

So, the cheese is ready to be marketed (see interview excerpt).

P1: Now in the afternoon I can start selling it by the pound.

I: Ready Mr. Angel, thank you very much for your kind attention and for sharing with us a little about your practice in the cheese factory.

P1: Thank you all.

# INSTITUTIONAL VISION OF THE EMERGING ETHNOMATHEMATICAL CONNECTIONS IN THE PRODUCTION OF COASTAL CHEESE

Given the applicability of mathematical content in the classroom and the generation of skills and competencies in the understanding of mathematical concepts, it is necessary to show a curricular approach that demonstrates the relevance of this research in relation to

Degree	BLR	Evidence of learning	Concept in daily practice	Mathematical concept	Application example
1°	6. Compare objects in the environment and establish similarities and differences using geometric characteristics of two- and three- dimensional shapes (curved or straight, open or closed, flat or solid, number of sides, number of faces, among others).	Create, compose and decompose two- dimensional and three-dimensional shapes, using clay, paper, sticks, boxes, etc. (p. 11).	<ol> <li>Basket</li> <li>"Zorrito"</li> <li>Block of cheese</li> </ol>	1. Rectangular prism 2. Elements, characteristics, properties and attributes of prisms	[] finds similar and different characteristics between the shape of the figures and the solids that compose them (see <b>Figure 29a</b> in <b>Figure 29</b> ).
4°	<ul> <li>4. Characterizes and compares measurable attributes of objects (density, hardness, viscosity, mass, capacity of containers, temperature) with respect to procedures, instruments and measurement units; and with respect to the needs they respond to.</li> </ul>	Recognize that to measure capacity and mass, comparisons are made with the capacity of containers of different sizes and with packages of different masses, respectively (liters, centiliters, gallons, bottles, etc., for capacity, grams, kilograms, pounds, arrobas, etc., for mass). (page 32)	<ol> <li>Liter jar</li> <li>Capacity of calambuco, tanks, yellow bucket</li> <li>Quantity of milk, "suero"</li> <li>Pounds of cheese dough per tank,</li> <li>"calambuco", bucket</li> <li>Rennet cap</li> </ol>	<ol> <li>Capacity measurement units (liter, cc)</li> <li>Dough measurement units (pounds)</li> <li>Solid capacity</li> <li>Weight of objects</li> <li>Volume of solids</li> </ol>	In social studies, Felipe is taught that it is advisable to select products that, in addition to being economical, offer recycling possibilities. What criteria are appropriate for selecting the best product from among several brands in terms of economy and recycling possibilities? (see <b>Figure 29b</b> in <b>Figure 29</b> )
5°	4. Justify relationships between surface and volume, with respect to dimensions of figures and solids, and choose the appropriate units according to the type of measurement (direct and indirect), instruments and procedures.	Construct and decompose flat figures and solids from established measurements (p. 39).	<ol> <li>Capacity of tanks, buckets, yellow buckets, baskets</li> <li>white cloth and its rectangular shape</li> <li>Fronts of boards, baskets, troughs, etc.</li> </ol>	<ol> <li>Capacity measurement unit with liter</li> <li>Calculation of areas and volumes</li> <li>Elements of solids: faces, edges, vertices, etc.</li> <li>Prisms, cones, semi- ellipsoids, parallelepipeds, truncated cones, etc.</li> </ol>	Using a 50 cm nylon string, make different rectangles. The perimeter of these rectangles is the same. Determine if their areas are equal. Determine if you can make rectangular boxes with different volumes, but in which the same amount of cardboard is needed to make their molds (see <b>Figure 29</b> .

Table 1. Institutional vision of the ethnomathematical connections present in the elaboration of coastal cheese for primar	y
basic education	-

BLR in the integration of ethnomathematical connections that emerge from daily practice and institutionalized mathematics (MEN, 2016).

Likewise, the functionality of this type of ethnomathematical connection in education (see Table 1) is recognized.

primary basic



Figure 29. Practical examples for working with primes and round bodies in primary education (extracted from MEN, 2016)

Examples of those offered in the BLR (MEN, 2016) are shown in **Figure 29**.

**Table 2** shows examples of BLR suggested for use in secondary education with some emerging concepts in the daily practice of cheese making.

Below are the examples offered in the BLR (MEN, 2016), see Figure 30.

**Table 2.** Institutional vision of the ethnomathematical connections present in the elaboration of coastal cheese for basic secondary and vocational secondary education

Degree	BLR	Evidence of learning	Concept in daily	Mathematical concept	Application example
7°	5. Observes three- dimensional objects from different points of view, represents them according to their location and recognizes them when they are transformed by rotations, translations and reflections (p. 55).	<ol> <li>Recognizes and interprets the representation of an object</li> <li>Represents three- dimensional objects when they are transformed.</li> </ol>	1. "Zorrito" 2 Tanks 3. Calambucos 4. Punch bowl 5. Baskets 6. Tights 7. "Canalete" 8. Liter jar/yellow bucket	<ol> <li>Generalities, specificities, properties, characteristics, elements and attributes of:         <ul> <li>prisms</li> <li>cylinders</li> <li>truncated cones and cones</li> <li>rectangles and circles</li> </ul> </li> </ol>	Observe an object from different points of view. Graphically represent the object if it is viewed from the front (front view), above (top view) and below (bottom view). Take photos of each view of the object and compare the images with the graphic representations made (see Figure 30a in Figure 30).
8°	5. Use and explain different strategies to find the volume of regular and irregular objects in solving problems in mathematics and	Use the relationship of capacity units with volume units (liters, dm3, etc.) in solving a problem.	<ol> <li>Liter jar</li> <li>Quantity of milk, "suero", dough; according to the object that contains them (tanks, jugs, buckets, baskets)</li> </ol>	<ol> <li>Capacity         measurement units in             <i>m</i><sup>3</sup>, <i>cm</i><sup>3</sup>, <i>dm</i><sup>3</sup>, etc.         2. Contrast of solids             with content, according             to capacity             3. Regular and         </li> </ol>	Associate the shape of the irregular object formed by a composition of regular figures, use these figures to

Table 2 (Continued). Institutional vision of t	he ethnomathematical connections present in the elaboration of coastal cheese
for basic secondary and vocational secondary	y education

Degree	BLR	Evidence of learning	Concept in daily practice	Mathematical concept	Application example
	other sciences (p. 61).			irregular solids 4. Volume calculations	calculate the volume of the irregular object with a reasonable approximation (see <b>Figure 30b</b> in <b>Figure 30</b> ).
9°	4. Identifies and uses relationships between volume and capacity of some round bodies (cylinder, cone and sphere) with reference to school and extracurricular situations (p. 68).	<ol> <li>Estimate the capacity of objects with round surfaces.</li> <li>Build round bodies using different strategies.</li> </ol>	<ol> <li>Tanks</li> <li>Press</li> <li>Punch-bowl</li> <li>Calambuco</li> <li>Amount of milk amount of "suero"</li> </ol>	<ol> <li>Cylinder: properties and attributes</li> <li>Cone and truncated cone: cross sections in solids, specificities</li> <li>Capacity of solids and capacity units: liter</li> </ol>	An industrial mechanic wants to check an estimate he has made in his work, regarding the relationship between volume. Justify whether the mechanic can check the estimate when constructing two metal parts as shown in the figure (see Figure 30).
Examp shown view: f shown object display views.	de: An observer viewed the containing the image from different points from view and bottom view shows the figure. As observed, shape configurations. Describe how of the packaging changes in each of the packaging changes in each of the packaging changes in each other shows of the packaging changes in the packaging changes in each other shows of the packaging changes in the packagin	The set of	<text><text><text><text></text></text></text></text>	A cylindrical container lied with water, an impletely submerged the displaced water is container that has been ced as shown in the	le: An industrial ic wants to check a e that you have made work, in regarding the ship between volume. if the mechanic when g two pieces metal like jown in the figure you ck the estimate.
Observe Graphicu the front (bottom each vie with the	an object from different points of ally represent the object if viewed (front view), above (top view) and b view). Take the photos corresponding wo of the object and compare the in representations graphics made.	view. from below ng to lages Find the arithmetic ratios between the different volumes of the boxes and the expression general for volume and total exterior area of each of them.	Compare the calculate the amount of water sp used and explains the procedures. Associates object formed for a co- uses these figures to object irregular to a rear	d volume with the volume of illed. Describe the procedures results and their respective to the shape of the irregular proposition of regular figures, calculate the volume of the sonable approximation.	ss and check how many is the contents of the cone- sed container the cylinder- bed one when filling them i different materials. Use the lit obtained by this redure to express the volume he cone in terms of cylinder ime.

Figure 30. Practical examples for working with three-dimensional figures in secondary education (MEN, 2016)

# DISCUSSION

We report the ethnomathematical connections present in the treatment of milk and the elaboration of coastal cheese, which is an influential social practice with great contributions to the local economy of Manatí and a transcendental activity to investigate mathematical objects (D'Ambrosio, 2014). Thus, ethnomathematical connections emerged that are included in the procedural phases of the practice from the collection and treatment of milk to the formation of cheese.

Given the presentation of the five mathematical thinkings suggested by the MEN (2006), spatial, metric, numerical and variational thoughts were also recognized as an opportunity to enhance mathematical skills from content grounded in student reality (Olivero-Acuña et al. 2022).

From spatial thinking and geometric systems, the two-dimensional geometric representation of flat figures (circle, rectangle), three-dimensional representation of solids and round bodies (parallelepiped, rectangular prism, cylinder, semi-ellipsoid, truncated cone) was evident, which showed ethnomathematical connections with artifacts such as the "zorrito", tanks, buckets, "canastillas", cheeses, "canalete", punch bowl, "pimpina", white cloth; thus allowing a cluster of places and geometric shapes useful for the detailed study of properties, attributes and graphic behaviors in the classroom. From the numerical thinking and the numerical systems, there was a great reception regarding representations in numerical lines, counting, arithmetic calculations, direct and inverse simple rules of three; which allowed to unite these concepts with artifacts such as the rod, the refractometer, quantity of rennet per milk, quantity of salt per dough, quantity of dough buckets per basket, quantity of milk calambuco per tank, quantity of sweet "suero" per tank, etc. Thus, numerical operational reasoning is enhanced from diversified contexts (Castañeda & Angulo, 2012).

Regarding metric thinking and measurement systems, conventional and non-conventional units of measurement were found to be fundamental and valued in other research (Rodríguez-Nieto, 2020; Rodríguez-Nieto et al., 2019; Morales-García & Rodríguez-Nieto, 2022), which represented the conversion of units of capacity and volume calculations in the use of rennet, syringe, tank capacity, buckets, calambucos and ratification of exact measurements according to the cheesemaker's postures and institutionalized mathematical calculations. Finally, variational thinking and algebraic systems were evident in the variation of quantities by the rules of three, thus showing the direct and inverse proportionality of the quantities of rennet, milk, "suero", and others, to show an accuracy in the procedures according to the reference taken.

Literature reports account for the universality of ethnomathematics as an opportunity to describe practices without borders (Santillán & Zachman, 2008). That is, what both contexts share in their practices can take on diverse meanings and uses such as ethnomathematical connections of meaning. A clear example is the gallon presented by Rodríguez-Nieto and Escobar-Ramírez (2022) that contains 7 pounds of green pigeon peas and the yellow bucket that contains 10 pounds of dough. As a result of this, Rodríguez-Nieto et al. (2022) focused their research on a geometric view of the shapes of cheeses from Chilpancingo-Mexico, establishing ethnomathematical connections with geometric shapes (mathematics in social practice = jars, cubes, circular) that account for the shape of the cheeses (institutionalized mathematics = cylinders) and their emphasis was on the commercialization of the cheese and its cylindrical shapes with congruent circular bases, thanks to the tube mold used for its production. This

research assumes a transversal view of the production of coastal cheese, with changes in its geometric shape.

The coastal cheese is made in "ampletas" or "baskets" in the shape of a rectangular prism, therefore, the structure of the "cheese block" is a rectangular prism. Thus, both investigations are assumed as social and educational study without borders, which allows us to approach mathematics itself from diverse contexts. But locally, Pérez-Ortiz et al. (2024) have bet on the ethnographic study of the commercialization of coastal cheese in retail or pieces with three types of cheese (square shape, circular shape, and rectangular shape).

A coherence is observed between the cheese from Barranquilla-Atlántico (rectangular shape) and the cheese made in Manatí-Atlántico (block or rectangular prism), which requires an amplified study in the commercialization of these cheeses with diagonal cuts in the blocks (retail) and distribution of parts in the formation of the concept of fraction applied in education to improve the understanding of this mathematical object through ethnomathematical connections.

This proposal shows relevance, since it has broken down various ethnomathematical connections that give rise to the concretion of mathematical objects categorized according to mathematical thoughts, which can be brought to the classroom to re-signify the mathematical educational practice (Fuentes, 2019; Rosa et al., 2017). Likewise, other research has concerted ethnomathematical innovation in social practices other than construction, crafts, weaving, etc., thus allowing a commitment to the study of the gastronomy of the communities to collect and rescue all these procedural phases that implicitly contain mathematical features useful to give life and coherence to mathematical content in school (Rodríguez-Nieto, 2021; Rodríguez-Nieto & Escobar-Ramírez, 2022).

In reference to the language and the meaning of the processes, this research has not sought to undermine the very essence of the practice in the treatment of milk and the elaboration of cheese; on the contrary, it has constituted an opportunity to rescue the traditions of the cheese makers of the municipality of Manatí and to enhance the implicit mathematics proper to them (Blanco-Álvarez, 2008). Likewise, undertaking this path under the applicability of mathematics education, supposes bringing to the classroom each and every one ethnomathematical connections of these and representations in order to ground the formation of mathematical concepts and objects with the very reality of nature (Pecharromán, 2013). In fact, a mathematical object makes sense when the student is capable of constructing it (MEN, 2006), and it can be quite interesting to teach, for example, the truncated cone with its properties, characteristics, geometric attributes and specificities through a punch bowl, which is an artifact proper to its context and daily use.

Given the previous considerations and meshing the ethnomathematical connections resulting from this study, it is essential to bet on the approximation of mathematical concepts from the reality of the context (Aroca, 2018a). In this way, the mathematical objects resulting from this study are ideal to be carried out in the classroom and give meaning to the task in the teaching and learning of mathematics from the generation of competences (MEN, 2006). The treatment of milk and the making of cheese are determining activities in the local context of Manatí and their reception in the classroom shows a different way of teaching mathematics from what is experienced in the gastronomic, cultural, social, and economic spheres, in order to decolonize knowledge and begin to re-signify from one's own (Leal, 2019; Tamayo & Méndez, 2021).

# CONCLUSION

This study highlights the ethnomathematical connections in the traditional practice of milk processing and coastal cheese production in Manatí, where mathematical concepts are linked to daily and cultural geometric activities. Through spatial thinking, two-dimensional and representations of threedimensional figures (such as circles, cylinders, truncated cones, rectangles, rectangular prisms) were identified in cheese production, facilitating the study of geometric properties. In numerical thinking, the use of arithmetic operations and rules of three to calculate precise quantities of ingredients was evident, connecting mathematical concepts with everyday artifacts such as refractometer and "varilla". Metric thinking showed the importance of conventional and non-conventional units in volume and capacity measurements in milk quantity and dough, while variational thinking addressed direct proportionality in ingredient quantities. From stochastics, random thinking determined the modeling connections for the creation of graphs and tables of values, which was useful in the organization of information such as the relationships between rennet caps and liters of milk. These connections allow contextualized and meaningful mathematics to be taught, promoting an education that is closer to the students' reality. Finally, this research has the vision of integrating ethnomathematical practices in the classroom, which not only improves the understanding of mathematical concepts, but also draws on local and community knowledge to contribute to students' understanding of their sociocultural environment. It should be noted that cheese making is a global daily practice, therefore, through external connections it can be contributed locally and internationally. For future research, it would be interesting to explore how the marketing and distribution of coastal cheese in different formats (such as cuts and fractions) can be used to teach mathematical concepts such as fractions, proportions and cost analysis, further integrating local economic

practices into the mathematical learning and financial mathematics education.

Author contributions: RRO-A: conceptualization, methodology, supervision, and writing-original draft & CAR-N, VFM, BMC-R, & FMR-V: formal analysis, validation, writing-original draft, writing-review & editing, and resources. All authors agreed with the results and conclusions.

**Funding:** This study is part of the projects, Proyecto de docencia codificado por DOC.100-11-001-18 (Universidad de la Costa) & Grant PID2021-127104NB-I00, funded by MICIU/AEI/10.13039/ 501100011033 & by "ERDF A way of making Europe".

**Ethical statement:** The authors stated that the participant or merchant of this research is the father of the first author, who has full access to the information and the endorsement to use it in his academic projects. The authors further stated that the merchant knows that this article has educational and not economic purposes, seeking to contribute to the teaching and learning of mathematics from ethnomathematical and mathematical connections.

**Declaration of interest:** No conflict of interest is declared by the authors.

**Data sharing statement:** Data supporting the findings and conclusions are available upon request from the corresponding author.

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