MODESTUM EURASIA Journal of Mathematics, Science and Technology Education, 2025, 21(2), em2573
ISSN:1305-8223 (online)
OPEN ACCESS Research Paper https://doi.org/10.29333/ejmste/15912

Colombian pre-service mathematics teachers' knowledge and didactic suitability in the context of the measurement

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Received 16 August 2024 ▪ Accepted 05 January 2025

Abstract

The research aim was to identify the didactic suitability criteria that emerge from the practice of pre-service mathematics teachers (PMTs) when they develop a workshop on measurement. The study was based on the theory of didactic suitability. The methodology was qualitative, developed in four stages: (1) a questionnaire was designed consisting of items related to concepts of the measure; (2) the participants, PMTs, were selected; (3) development of a workshop in which a questionnaire-based task is implemented to collect data within the context of participant observation, and (4) the data were analyzed based on the theoretical foundation and didactic suitability indicators were proposed to teach the measure. The results show the importance of agreeing on criteria and components and establishing indicators of didactic suitability specifically in terms of cognitive, epistemic, interactional, mediational, affective, and ecological.

Keywords: measurement, didactic suitability, pre-service mathematics teachers, mathematics education

INTRODUCTION

In various curriculum frameworks, it is stated that the activity of measuring, measurement, and the measure are important for the teaching and learning of mathematics at different educational levels (Ministerio de Educación Nacional [Ministry of National Education] [MEN, 2006]; National Council of Teachers of Mathematics [NCTM], 2000). In fact, since its beginnings, humankind has had the need to use its body to carry out measurement processes, considering non-conventional units of measurement that over time became standardized (Castaño, 2015; Kula, 1980; Rodríguez-Nieto & Alsina, 2022). Likewise, measurement provides opportunities for students and/or teachers to learn and apply institutional mathematics and those related to the context, where numerical operations, the use of geometric notions, variation, and statistics are evident, which are connected with other subjects such as arts education, social sciences, among others (NCTM, 2000).

In the field of mathematics education, measurement has inspired multiple authors to research and provide solutions to problems associated with this mathematical object, which has been addressed from various theoretical and methodological perspectives. For example, Godino et al. (2003) presented a study on measurement from the school perspective; Szilágyi et al. (2013) conducted research on the measurement of lengths related to levels of thinking; Codina et al. (2017) demonstrated some potentialities and difficulties in handling didactic sequences for the magnitude of surface area; Alsina and Salgado (2018, 2019) examined, from a realistic perspective of mathematics education, the historical epistemological element of measurement and practices associated with measurement; Vanegas et al. (2019) provided an analysis of the structure and functioning of a teacher's class who works on measurement with 12- and 13-year-olds, according to the proposed model of didactic-mathematical knowledge and competencies of the onto-semiotic approach (OSA); and the study by Rubio and Vanegas (2018), which

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Contribution to the literature

- The exploration of mathematical didactic knowledge (MDK) of pre-service mathematics teachers (PMTs) when solving problems about the concept of measurement.
- The creation of specific criteria of didactic suitability around the concept of measurement.
- The importance of generating or proposing didactic suitability in teacher training, specifying the use of media, mathematical concepts, participation, argumentation and especially the assessment of the close cultural context of PMTs.

configured and validated an instrument for characterizing the level of understanding of length measurement learning in children aged 6-8 years.

In this context, we have outstanding research focused on measurement based on ethnomathematics which favors the teaching of measurement systems from a sociocultural and complementary perspective for the tasks promoted in curricular materials (Ávila, 2014; García-García & Bernandino-Silverio, 2019; Gerdes, 2013; Rodríguez-Nieto, 2021); These authors agree that non-conventional units of measurement such as the quarter, the span, the fathom, the elbow, the finger, the foot, among others involved in different daily practices, should be valued and used.

Although the content of the measure has been studied by various authors with their respective theories, it was also recognized that students, PMTs and in-service mathematics teachers show difficulties when using this mathematical object to solve intra-mathematical and extra-mathematical problems (Sala-Sebastià et al., 2022). This problem can be caused by conceptual errors and inconsistencies during an educational practice, for example, Pla-Castells et al. (2021) identified that, in a second-year course of future primary education teachers, errors prevail to master concepts and processes when modeling length measurements is developed. Furthermore, errors are reflected in

(1) the omission of complex necessary elements that makes it difficult to mentally represent the real situation in the construction of a model;

(2) the lack of rigor when selecting the data necessary for the problem statement;

(3) inadequate management of the scale that relates the measurements of a plane and the real measurement of an object (enlargement-reduction);

(4) incorrect use of the equal sign, which causes erroneous expressions of equivalence;

(5) PMTs tend to favor the use of the rule of three in the search for the answer to any mathematical problem, engaging in the mechanization of processes to solve problems;

(6) there is evidence of application of recently taught concepts and processes, regardless of whether they are relevant or not for an efficient solution to the problem and finally,

(7) little argumentation in the implementation of arithmetic calculations, leading to a weak validity of the application of processes.

In this line, measurement has been of interest in curricular materials, for example, in textbooks the treatment of measurement, measurement and units of measurement is reported more frequently in activities where the use of arithmetic operations is required and little use of the conceptual and pragmatic aspects that could favor students' understanding inside and outside the classroom (Chamorro, 2003). Also, in Mengual et al. (2017), the conceptual and procedural aspects associated with the measurement remain in the background, given that the use of algorithms predominates. During the implementation of the practices of some PMTs in the mathematics degree program, the evasion of the surface coating strategy to solve area problems was observed, preferring to insistently apply formulas to calculate the solution in a complicated and unattainable way for the students.

Given the complexity evident in the content of the measure, it is necessary to address the aspects that affect the practice of teachers in service or in training because it is a problematic and at the same time motivating factor to be investigated and strengthened to implement it in the classroom. These types of studies are framed from the reflection of teaching practice based on the didactic suitability criteria (DSC) of the OSA. It should be noted that, with these tools, mathematical practice in teacher training has been analyzed, including the use of components and indicators that direct the teacher's reflection and allow the development of competencies to assess the teaching and learning processes of mathematics related to specific mathematical contents or processes (e.g., Breda, 2020; Burgos et al., 2020; Esqué de los Ojos & Breda, 2021; Giacomone et al., 2018; Hummes et al., 2023; Ledezma et al., 2021; Ledezma et al., 2023; Sánchez et al., 2022; Seckel et al., 2019, 2021, etc.). The reflection that is suggested is fundamental because it leads to the appropriation and management of the complexity of the mathematical objects that the teacher must use in his mathematical practice, as well as the design and application of didactic sequences so that teachers guide their class with students in an ideal

manner (Breda et al., 2018; Calle et al., 2021; Godino et al., 2019; Sala-Sebastià et al., 2022).

One of the main difficulties in measuring areas by teachers and students is the use of the algorithm area = base × height for all geometric figures, without considering their particular characteristics, instead of using the idea of covering the surface with a unit. Another common difficulty is the confusion between the concepts of perimeter and area. This may be due to the inability to differentiate between them or to the mistaken belief that rectangles with equal perimeters must have the same area. These difficulties have been identified in various research on mathematics education (Asil-Güzel, 2018; Asil-Güzel & Yeşildere-İmre, 2024; Battista, 1982; Clements & Stephan, 2004; Passelaigue & Munier, 2015).

Particularly, there is a problem with the understanding of concepts such as measurement, length, distance, among others. For example, Sánchez-Matamoros et al. (2018) provided valuable information to the research that invites further research on this topic, given that: the main problem identified in the information provided is related to the difficulties faced by EPM (teacher practice students) in developing an effective learning path in teaching magnitude, length and its measurement, and, specifically, in acquiring the "look professionally" competence to recognize and understand children's mathematical thinking. This problem is broken down into several key aspects:

1. The implementation of a learning trajectory is a complex process that goes beyond the simple implementation of a teaching module in the context of a subject. Only a small number of EPM have managed to complete this process, suggesting that the challenge is significant.
2. Many EPM have failed to correctly recognize certain key mathematical elements, such as conservation of magnitude or fundamental aspects of measurement, which has limited their ability to develop deep understanding in children. This is reflected in the inability of EPM to complete the instrumental action schemes necessary to advance in their teaching competence.
3. The process of developing instrumental action schemes is progressive and the second scheme has not been reached by many EPM, indicating that they have managed to identify some mathematical elements, but have not managed to establish a significant relationship between these and children's mathematical thinking, crucial for understanding and learning.
4. A group of EPM has designed tasks that are not based on a deep interpretation of students' understanding, which could generate a cognitive leap that hinders children's learning. This shows that many EPM still fail to develop tasks that effectively favor the progression in children's mathematical understanding.
5. EPM who fail to use the learning path effectively often fail to establish a connection between their knowledge and children's mathematical thinking. This reflects a critical difficulty in advancing in the competence of "looking professionally", which is essential to correctly interpret children's responses and assess their understanding. Along these lines, Costado Dios (2023) applied training experience with pre-service primary education teachers on geometric concepts and measurements, where it was important to establish connections with real-life situations. However, these types of activities should continue to be carried out because some pre-service teachers require mathematical, geometric and didactic knowledge to improve their future teaching practice.

After reviewing the specialized literature on the topic of interest, we recognized the existence of multiple problems of PMTs and in-service teachers that could rest and unfavorably influence student learning. Therefore, the objective of this research is to identify the DSC that emerge from the practice of PMTs when they develop a workshop on measurement.

THEORETICAL FOUNDATION

The theoretical foundation of this research is part of the set of analysis tools, proposed by the OSA, defined by Godino et al. (2021) and Morales-Garcia et al. (2022) as a theoretical, modular and inclusive system to strengthen teaching and promote learning of mathematics which, according to Gusmão and Font (2020) and Sala-Sebastia et al. (2022), guides what the teacher should do in designing tasks.

This analysis is based on MDK that according to Godino et al. (2020) and Morales-García et al. (2022) seeks the abstract construction of the mathematical knowledge of the learning subject, when the teacher starts from the analysis of the student's previous experiences and the understanding of social, cultural and political aspects of the context. Furthermore, through the theory of didactic suitability, it agrees on criteria for the study and monitors the practice, based on the design of suitability indicators.

Mathematical Didactic Knowledge

In Breda et al. (2023), it can be observed how it is possible to infer characteristics of the MDK of PMTs from the development of tasks carried out during practice. This analysis is guided by the interpretation and characterization of the mathematical dimension, the didactic dimension and the meta-didactic-mathematical dimension. In the first two, the teaching-learning process of mathematics is described and explained and in the

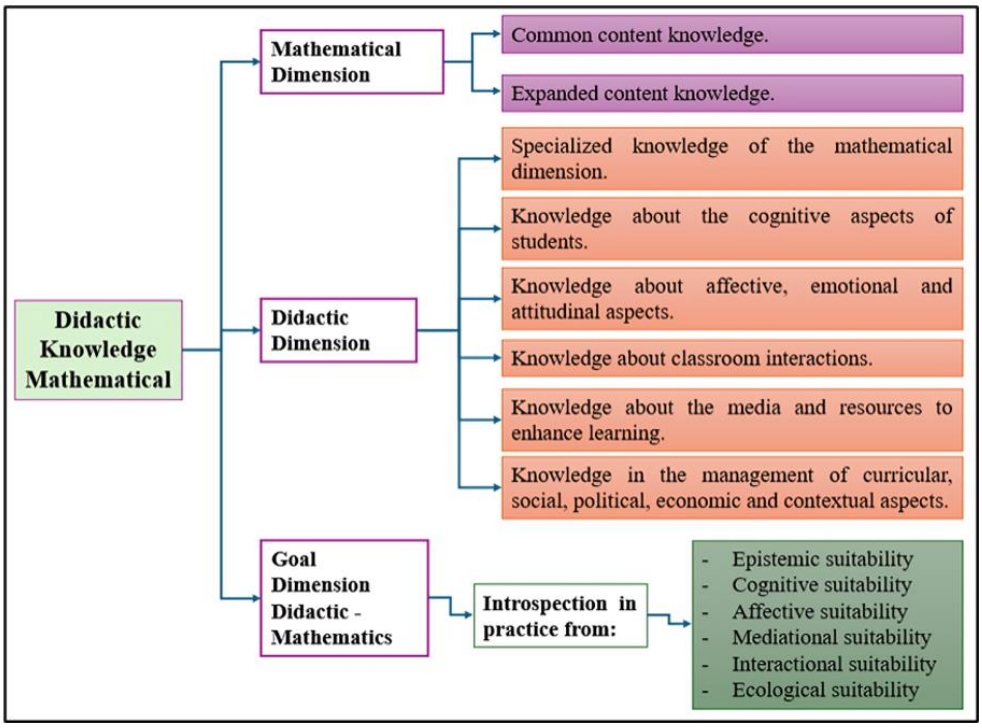


Figure 1. Categorization of MDK (Breda et al., 2023)

third, the epistemic, the cognitive, the interactional, the affective, the mediational and the ecological are valued in practice.

Regarding the three dimensions, Pino-Fan and Godino (2015) point out that

> the mathematical dimension has 2 subcategories: Common knowledge of the content, which refers to knowledge sufficient to solve a problem or task proposed in the mathematical curriculum, that is, knowledge about a mathematical object; and the expanded knowledge of the content, alluding to the knowledge that the teacher must have about mathematical notions, to propose new challenges, here personal meanings are mobilized to institutional meanings.

The didactic dimension considers the following subcategories: specialized knowledge of the mathematical dimension; knowledge about the cognitive aspects of students, knowledge about the affective, emotional and attitudinal aspects; knowledge about interactions in the classroom; knowledge about the means and resources to enhance student learning; and the knowledge that influences the management of learning based on curricular, social, political, economic and contextual aspects.

The meta-didactic-mathematical dimension begins with a teacher who rationally reflects on his or her judgments, decision-making, beliefs and routines, in order to pursue best practices. Introversion is carried out under the proposal of DSC reflected from the teaching-learning process of: The epistemic, the cognitive, the affective, the mediational, the interactional and the ecological. **Figure 1** shows the categorization of MDK.

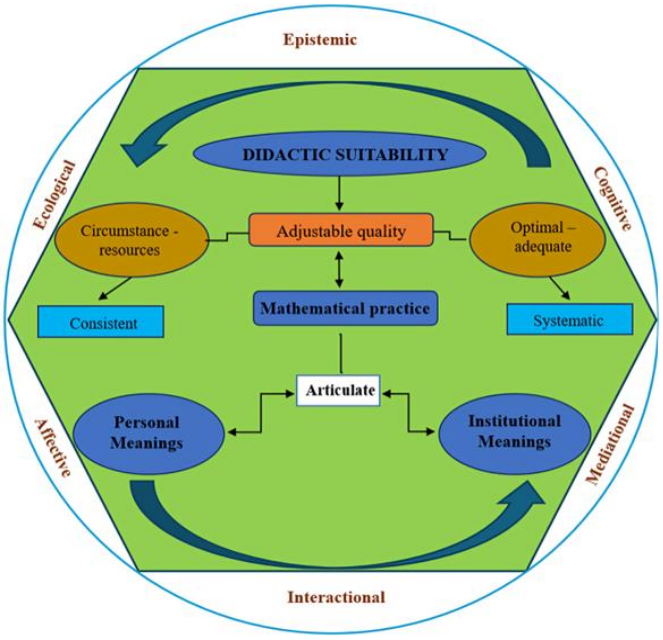


Figure 2. Reflection on didactic suitability (Breda et al., 2023)

Didactic Suitability Criteria

For the purposes of this research, didactic suitability, represented in **Figure 2**, is defined as the adjustable quality of a teaching-learning process (or a part of it), which can be qualified as optimal or adequate, considering the circumstances and resources for the achievement of the personal meanings achieved by students and the institutional meanings intended or

Table 1. Criteria and components of didactic suitability

Criteria	Components
Epistemic	IE1–Errors made by PMTs when developing a measurement task. IE2–Ambiguities proposed by PMTs during the performance of a measurement task. IE3–Richness of processes used when solving an oversized task. IE4–Representativeness of meanings used by PMTs on the topic of measurement.
Cognitive	IC1–Students' prior knowledge on the topic of measurement IC2–Curriculum adaptation to the individual differences of the student on the measurement topic. IC3–Approach to student learning on the topic of measurement. IC4–High cognitive demand according to the mental operations carried out by students when addressing the topic of measurements.
Interactional	II1–Teacher-student interaction during the communication process in classroom practice on the topic of measurement. II2–Interaction between students during the communication process in classroom practice on the topic of measurement. II3–Autonomy to carry out tasks on the measurement topic II4–Formative evaluation with constant feedback on the learning of the measurement topic.
Mediational	IM1–Resources and materials that facilitate learning about measurement. IM2–Number of students, schedule and classroom conditions during the practice. IM3–Times for the development of tasks with students on the measure.
Affective	IA1–Interests and needs of students when addressing the topic of the measure. IA2–Attitudes observed in students when performing a measurement task. IA3–Emotions aroused by students during the development of a measurement task.
Ecological	IEC1–Adaptation to the curriculum according to the context in which a measurement task is developed. IEC2–Intra and interdisciplinary connections of the purposes of a measurement task. IEC3–Socio-labor utility of the measurement topic. IEC4–Didactic innovation that leads to measurement learning.

Note. Information taken from Breda et al. (2017), Ledezma et al. (2021), and Morales-López and Font (2019) and adapted by the researchers

implemented (Breda et al., 2018; Cotrado et al., 2022; Godino et al., 2006).

To do this, it is necessary that the mathematical practice involved in a highly suitable instruction process articulates, in a coherent and systemic manner, the following criteria (Font et al., 2010):

1. **Epistemic suitability:** It represents the complexity of institutional meanings according to the political, cultural and interdisciplinary context, for good mathematics teaching (Ledezma et al., 2021).
2. **Cognitive suitability:** Zone of adaptation to the instructional process, where personal meanings are close to those intended. Effort in the teaching-learning process, so that the student learns, where previous concepts prevail (Godino 2013; Ledezma et al., 2021).
3. **Interactional suitability:** Didactic configuration of teacher-student and student-student interaction, to identify and resolve potential semiotic conflicts, promotes communication, reflection and shared argumentation (Cotrado et al., 2022).
4. **Mediational suitability:** Selection of material and temporal resources to achieve adequate classroom practice, especially those contents that show greater learning difficulty (Breda et al., 2018; Cotrado et al, 2022).
5. **Affective suitability:** Involvement of the student's interests and motivations, to achieve better learning. Based on situations that allow emotions, attitudes, beliefs and values to be identified in mathematical practice (Breda et al., 2018; Cotrado et al., 2022; Ledezma et al., 2021).
6. **Ecological suitability:** Adequacy of the curricular reference, in accordance with the educational project of the center in question, based on the connection with other content and elements close to the students (Breda et al., 2017; Ledezma et al., 2021).

Components of Didactic Suitability

The indicated criteria, as a gradable quality, are supported by observable didactic suitability indicators, oriented by components, which specifically monitor the teaching-learning process during the development of tasks (Breda, 2020). The operation of the set of criteria and components can be seen in **Table 1**, which guides the description of indicators of didactic suitability, according to the task followed.

The monitoring of the development of a task in class provides truthful and effective information on the language, concepts, propositions, procedures and arguments, which lead to structured reflection, to provide better planning, development and assessment of pedagogical practice (Rodríguez-Nieto et al., 2021; Vanegas et al., 2019). If this monitoring is carried out

guided by the criteria set out in **Table 1**, it is possible to establish clear relationships that favor improving teaching and learning processes in a given context.

Structured reflection, which revolves around the criteria and monitoring components agreed upon in a Task, allows for subsequent analysis which, when related to each other, leads to the description of indicators of didactic suitability.

METHODOLOGY

The methodology of this research is qualitative (Cohen et al., 2018), and it is developed in the following stages:

1. Design of a workshop in the form of a questionnaire, composed of a task related to the meaning of measurement, measurement units, measurement pattern, and estimation. In addition, it includes fundamental aspects of an artifact and the usual measurement situations in the investigated context (rural and urban)
2. Selection of participants (PMTs)
3. Development of a workshop in which a questionnaire-based task is implemented to collect data within the context of participant observation
4. Analysis of the monitoring or support of classroom practice during the execution of the task and its connection to the theoretical foundations of this research, particularly with regard to didactic suitability.

The fourth stage of this research leads to a consensus among researchers on the criteria and components of didactic suitability from a theoretical perspective, which subsequently favors the analysis of a measurement task, to provide suitability indicators that guide its teaching.

Questionnaire

A workshop consisting of six tasks was designed with the aim of developing teaching skills. The first set of questions explores the knowledge that PMTs have about measurement, units of measurement, and measurement patterns, with the purpose of identifying and understanding their meanings. The second set of the instrument includes a table that distinguishes the units of measurement for length, time, mass, and capacity used in the context of the topic being studied, and asks for a description of their respective use, ensuring the recognition of appropriate resources that facilitate the understanding of measurement concepts. Finally, the third set of questions invites participants to form teams and reflect on the types of problematic situations that interest and motivate them, observing the dynamics of interaction during collaborative work with their colleagues.

A fourth group of questions seeks to recognize which are the effects that promote interest in learning about the measure in the context, evoking what is comfortable and pleasant for the learner. The penultimate collection of questions leads to the design of an ideal problem situation, related to the measurement and interest of the learner in the context studied. Finally, the curricular elements to which the measurement topic can respond in a specific grade are questioned and the proposal of a class plan on measurements is requested from PMTs, in real time and in accordance with the previous reflection.

Participants and Context

The implementation of the programmed task involved the participation of twenty students who are in the IX semester of the mathematics degree program at the University of the Colombian Caribbean and the accompaniment of two of the researchers of this study. This group of PMTs belong to the south of the Atlantic and surrounding municipalities of the departments of Bolívar and Magdalena.

Workshop and Data Collection

During the initial period of the practice, one of the researchers presents the task that is going to be treated and shows the questions that must be clarified during class, activates memories about the topic and mentions the importance of recognizing previous concepts to accommodate the new learning and points out the progress that is intended during the class. The other researcher records the dialogues, takes records and supports with the management of the materials.

Then, each student is given the questionnaire that will guide the classroom practice and is invited to respond first during a space individually to the groups of questions #1, #2, and #3, then a dialogue is established between the researchers and PMTs oriented according to the order of the questions, in which the students' answers are heard first and then feedback is given. For groups of questions #4, #5, and #6, teams are formed, and the same procedure is applied, including observation by the second researcher on the way the students interact.

Finally, the researcher points out the importance of the topic discussed and highlights the findings found, the patterns and artifacts used, and anecdotes told during the practice by the students that are significant for learning the topic and facilitate a better approach to knowledge. Highlighting the context and culture in which they operate and that can favor the teaching process.

Analysis of Data

For the study of each group of questions in the questionnaire, the responses of the PMTs and the feedback from one of the researchers, carried out during

classroom practice, are listed in organized tables which leads to a subsequent analysis of the answers, mentioning the question, the answer, the feedback and the subsequent analysis.

Taking the previous analysis as ideas that guide the criteria and components first agreed upon among the researchers, according to the planned questionnaire, indicators of didactic suitability are established, under an argument based on the OSA and complex thinking, when a task is analyzed of measurement, presenting criterion, component and suitability indicator.

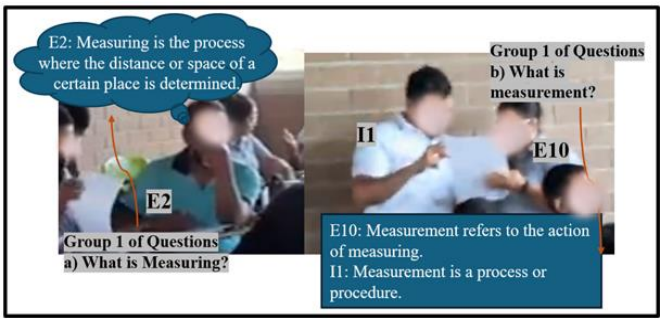


Figure 3. Personal meaning of the measuring and measurement (Source: Authors' own elaboration)

RESULTS

During this stage of the research, the didactic suitability is analyzed, especially of a PMT (E2) of the IX semester of a bachelor's degree in mathematics, among 19 others and the guiding teacher (researcher 1: I1), during the development of a measurement context task. For this, we reflect on the complex dialogic that is carried out during pedagogical practice, with the intention of achieving an optimal or adequate adjustable quality of the teaching-learning process (Breda et al., 2018; Cotrado et al., 2022), the which revolves for this research, around epistemic, interactional, mediational, affective and ecological adequacies. Next, the responses obtained during the implementation of the questionnaire are analyzed according to the didactic suitability proposed by the OSA.

Epistemic Suitability, Analyzes Meanings About Measurement

This first group of questions leads us to reflect on the knowledge that PMTs bring to the teaching-learning process of measurement, as can be seen in **Figure 3**.

The analysis of this group of questions is carried out with the help of **Table 2**, which relates the personal meanings of a PMT (E2).

Table 2. Group of questions of the implemented questionnaire with PMTs

Group 1. Based on your knowledge, answer the following questions					
1. What is measuring? 2. What is measurement? 3. What do you understand by unit of measurement? 4. What units of measurement do you know? 5. What is the measurement standard? 6. What is estimating?					
Group 2					
a. Present in the following table the type of standard or unit of measurement used in your context, agree with the attribute and briefly mention how it is used.			b. Complete the following table, pointing out the artifacts you use in your home or the environment where you live to measure the following units, then make a short description of how they are used.		
Kind of measure	Typical unit or pattern of the context, which is used to measure.	Description of how to use the pattern or mentioned unit	Kind of measure	Typical unit or pattern of the context, which is used to measure.	Description of how to use the pattern or mentioned unit.
Length Time Mass Ability			Length Time Mass Ability		
c. What tools could you use to address the issue of measurement?					
d. If they suggest using web tools or software to develop the topic of the measure, which one(s) would you use? Give an example					
Group 3: Answer the following questions according to your interests and motivations			Group 5		
1. What type of problem situation, related to measurement, would you like to work on? 2. What type of measurement situations do you relate to or have you related to? 3. Form groups of three participants (PMTs) to share the answers obtained (in the previous questions) with your classmates and provide an example that, for you, has contributed to your training as high school students or in your training process as mathematics teachers.			1. How do you represent the measure and measurement processes? 2. As a PMT, how would you teach the concept of measurement to your students? 3. Design a problem that involves the concepts of measuring, measure, unit of measurement and solve it in detail.		

Table 2 (Continued). Group of questions of the implemented questionnaire with PMTs

Group 4: Share according to the following questions	Group 6
1. What type of interactions allows you to have comfortable and pleasant learning at the time of work the concept of measuring? 2. What are the conditions (context or daily situation) surrounding the measurement processes in which you consider that do you learn better?	1. What elements of the curriculum or study plan would you use to address the concept of measuring? 2. Propose an example of class planning where you involve the concepts of measuring, measure, unit of measurement and considers the school grade.

Note. Complete questionnaire in https://drive.google.com/file/d/1m5s1BziF5ZAgJeFXeyQ5vtJGj6_l0lyB/view?usp=sharing

Table 3. Analysis of the knowledge with which PMTs arrive at the teaching-learning process of the measure

Questions	Answers E2	I1 feedback	Posterior analysis
What is measuring?	Process where the distance or space of a certain place is determined.	It is comparing a unit of measurement with a magnitude.	AP101–Confuses the concept of measuring with that of measurement, referring to process.
What is measurement?	They are the established parameters or results obtained in the process to obtain the measurement.	It is a process or procedure.	AP102–Limits the meaning of measuring distance and space. AP103–It is close to the concept of measurement, because it is understood as a process, similar to the institutionally established one.
What do you understand by unit of measurement?	They are the different types of measurements used to measure a space or distance.	It is a conventional or non-conventional quantity that you adopt to establish a measurement, through a precision process.	AP104–Understand the existence of various units of measure used to measuring. That is, it is inferred that it distinguishes the measures.
What units of measurement do you know?	Meter, kilometer, centimeter, inch, hectare. Fourth, a foot, a rope, the height of the navel are non-conventional. *A piece of land that is equal to one hectare. A nail, half a nail. which is a centimeter.	The height of the navel, the yard, the elbow, a fish hand, a rope of land. By approximating a square of land to the value of a hectare, connections are made. One nail, half a nail, the fourth.	AP105–Recognizes the meter, kilometer, centimeter, inch, hectare as conventional units of measurement. AP106–Point out the fourth, the foot, the cabuya, the height of the navel and the nail as non-conventional measures. AP107–The I1 intervention serves to understand mathematical connections and establish relationships between conventional and non-conventional units of measurement.
What is a measurement standard?	It is a guide measure The idea is inferred: A hand of fish is 4 and a hand of corn is 5.	It is a measure that standardizes a certain group or place. The pattern is something that is standardized and discussed commonly in a certain community, to establish a measure.	AP108–It ignores which pattern is relative to the agreement of a group or place, although it presents the example, it fails to conceptualize the situation. AP109–Take advantage of the example of the fish and corn hand as an example to understand the meaning of pattern.
What is estimating?	It is the approximation or determination of a measurement.	An approximate measurement, what one thinks it measures, almost exact	AP110–A similarity is observed between personal and institutional meanings, regarding the approximation of a measure.

Note. The meanings obtained correspond to the answer given to group of questions #1 by student E2

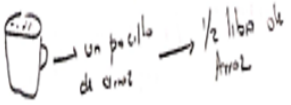
The feedback given by the researcher (I1), during the development of this task, see **Table 3**.

According to the analysis carried out, the concepts expressed by the PMT about measuring, measurement and unit of measurement can be recognized as confusing in the definition of what it means to measure and the omission of elements in his response, which restricts the concepts to the particularity of distance, forgetting other magnitudes, such as mass, time, temperature, among many others and overlooking, attributing the concept of Pattern to a specific place group. Reflecting the suffering of errors pointed out by Pla-Castells et al. (2021), referring to the forgetting of some elements that represent the complexity in reality of the object studied and the lack of rigor when analyzed (IE1).

On the other hand, the previous analysis becomes a fundamental input for the mathematical connection, starting from non-conventional measurements, such as the measurement of the nail, the fourth, the navel, the cabuya among others, to arrive at conventional measurements, such as the centimeter, decimeter, meter or hectare. They allow clear communication to be established between the teacher and the students, providing a path with appropriate language to approach new knowledge, according to the personal meanings of the students.

Figure 4. Agreeing on institutional meanings of measurement (Source: Authors' own elaboration)

Table 4. Representation of different measures, in the context of PMTs

Questions	Answers E2	I1 feedback	Posterior analysis
How do you represent measure and measurement processes?	 A small cup of rice is equivalent to half a pound of rice.	The measure of a hand-span, of two hand-spans of three hand-spans or the measure of five hand-spans. Now how did you make that measurement model? The ones that E5 was doing right now (simulates equal parts with a specific distance between the hands, makes the representation with gestures) these are processes. In conversation with E2, very good, you place the bowl of rice and its equivalent.	AP501–It is considered to measure with the rice well, to adapt to conventional weight measurements.
As a PMT, how would you teach the concept of measurement to your students?	Through a field trip taking students to work or relate peasant concepts to mathematical ones.	The objects that are around the students. He considers the intervention of E15 excellent, who commented, the example of the measurement from the navel to the floor, where you start with a non-conventional measure and then formalize it and work with the conventional measure. First of all, the environment and environment where you are going to work must be defined. It must be considered that technology does not only refer to a computer, but it also refers to various artifacts, for example a leaf, a chair, a totem, a shoe. I defend myself with the technological tools that have been evolving over time.	AP502–Present social utility to the topic of measures through the presentation of problem situations that arise from peasant work.
Design a problem that involves the concepts of measuring, measurement, unit of measurement and solve it in detail.	It is proposed: We will go to the soccer field, we will stand at one end and we will measure the time in which 5 students first run and then walk around the field. Once the activity is finished they will be asked: In this case, what have we measured? Finally, we will realize that: • Distance • Time • Speed	It is usually interesting to do a classroom practice on the students' soccer field, surely they will be pleased and motivated, they also learn to measure according to realities and this can be reasons for them to develop tasks in free time, such that they strengthen the measurement learning. In addition, other types of measures can be applied.	AP503–Posing problem situations on and on the soccer field awakens emotions in students when solving a measurement task.

Cognitive Suitability, Recognizes Representations About the Subject of the Measurement

During this stage of the workshop, the way in which the PMTs represents the measure, and its processes is questioned, recognizing some characteristics that make up the institutional meanings and favorable problems about the measure. In addition, we reflect on the most appropriate way to teach this topic (**Figure 4**).

Table 4 investigates the way in which the PMTs represents its meanings, how they teach it and what problem situations he uses for the topic of the measurement. Then, he reflects on it and highlights the particularity of the context.

In the social and cultural context studied, measuring the amount of rice used with a well, recognizing the usefulness of the measurement in peasant work and addressing a problem situation on the town's soccer field, is considered a very common action for many of the inhabitants of the Municipality of Suan and produces interest in the subject taught, since it provides ideas that can be used to propose situations that propose interdisciplinary work.

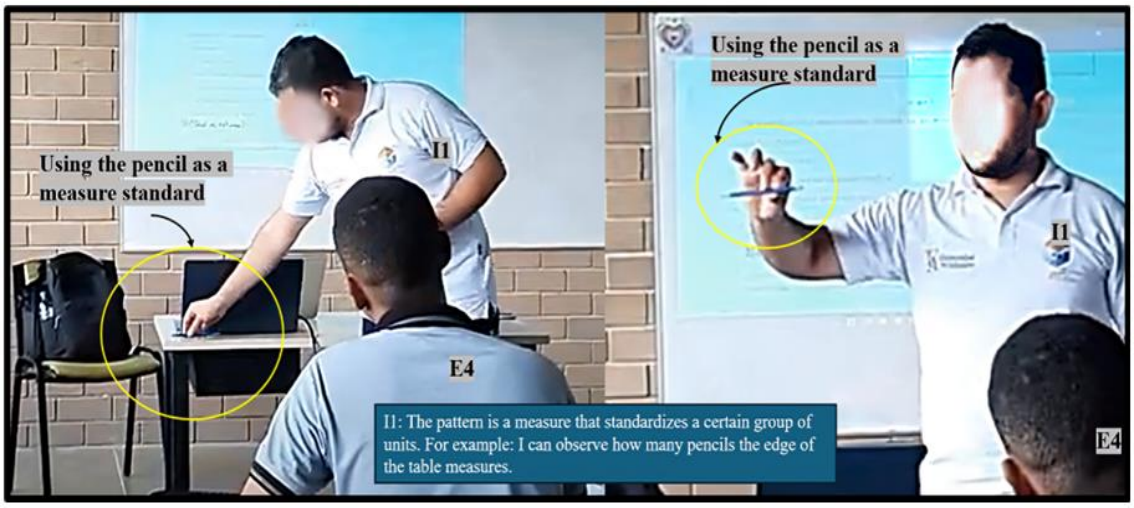


Figure 5. Pencil as a measuring standard (researcher 1 uses a pencil as a template to measure the length of the desk) (Source: Authors' own elaboration)

The previous situations are part of a fundamental component that involves various mathematical processes and objects, as pointed out by Calle et al. (2022), which emerge from practice and are part of the complex spaces proposed by Font et al. (2013), where The mathematical meanings start from a known situation, the context, to provide a better understanding and development of competence on the subject of measurement, providing a clear path for the construction of institutional meanings as proposed by Ledezma et al. (2021).

In this stage of the workshop analysis, the way in which the PMTs represents the measure, and its processes is questioned, recognizing some characteristics that make up the institutional meanings and favorable problems about the measure. In addition, we reflect on the most appropriate way to teach this topic.

Mediational Suitability, Recognizes Patterns and Context Instruments

Listening to what resources are available to PMTs in the context studied provides a reality of the concrete material with which students can construct mathematical meanings and facilitate adequate planning of teaching resources that promote better learning about the measure and guides the institutional meanings. In **Figure 5**, we see a pencil as a standard measuring.

Table 5 shows the measurement units or patterns used in the investigated context (E2) and recognizes them from the feedback of the researcher (I1), finding the particularity used in the place, to facilitate adaptation to the new learning of extent. The identification of elements that surround the learning subject guides appropriate pedagogical and didactic planning for those who teach, because it allows the use of appropriate language to

Table 5. Analysis of patterns and measurement instruments used in the context

Questions	Answers E2	I1 feedback	Posterior analysis
Present the type of standard or unit of measurement used in your context, according to the attribute and briefly mention how it is used.	The cabuya, a hand-span, a nail, half a nail: Used for planting the corn crop. Also, E2 mentions that he measures time through songs he listens to on his phone when he travels from his home to the university. 4 tobaccos: When burning 4 tobaccos the time to reach a certain place is approximately 1 hour. 1 load: equivalent to 2 bundles of corn, which is equivalent to 40 hands of corn. Where 80 hands of corn is equivalent to 200 ears. For example, the saliva that grandfather used to spit, he said, I'm going to spit here and before it dries you must be here. There was an estimate of the time it took to go to the store and come back.	When he leaves his house, he puts on his headphones and plays 3 to 4 songs to get here to the university, so, that's a unit of what? Time, right. What happens? The unit of measurement is a song. What's non-standard is how long it lasts, then. Will it be conventional or non-conventional? It's non-conventional. But then you will reach an average, where you say, one took 3, the other 4:30 ... That is your pattern, when you standardize something, that you carry in your pocket or briefcase. That's your pattern.	AP201–Use the rope and the earth quarter to approximate the meaning of area measurement units. AP202–Use the Nail to understand the centimeter as a measure of length. AP203–The use of burning tobacco, listening to a certain number of songs and the grandfather's saliva facilitate communication about him, meaning of unit of time. AP204–The usefulness of the hand of corn within educational language contributes to the approximation of the meaning of the unit of weight.

Table 5 (Continued). Analysis of patterns and measurement instruments used in the context

Questions	Answers E2	I1 feedback	Posterior analysis
If they suggest using web tools or software to develop the measurement topic, which one or which would you use? Give an example.	GeoGebra	GeoGebra, like other resources usually used such as the subway and the cell phone, currently help us take measurements.	AP205–Use the tape measure as a didactic mediation instrument for the topic of length measurements. AP206–Use cell phones as a teaching resource for learning about the topic of measurements. AP207–It is possible to use the GeoGebra Software to teach metric processes.

Note. The answers in the second column were given by student E2 for group 2 of questions

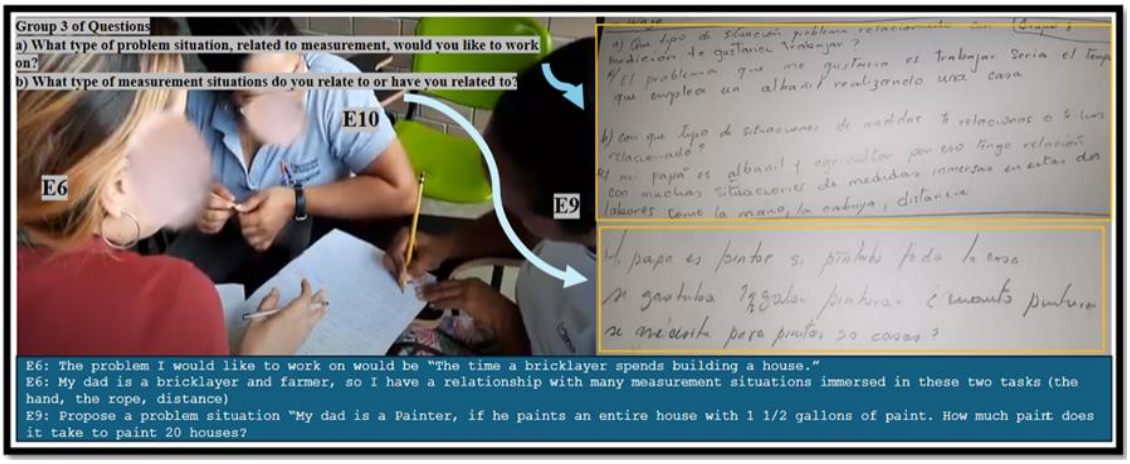


Figure 6. I share as a team about the measure (Source: Authors' own elaboration)

approach new knowledge (Calle et al., 2022; Cotrado et al., 2022). Use the term cabuya or cuarterón to approach the calculation of a certain area of land with non-conventional measures, contributing to the construction of the meaning of area.

Furthermore, the contextualization of time magnitudes, with the example of an adult who uses burning tobacco, listening to a certain number of songs or spitting on the ground, to introduce the internationally agreed management of this type of unit. We can also include the knowledge that the population has about the marketing of corn hands to establish didactic strategies that favor the teaching of weight measurements. Likewise, it is interesting to know the artifacts that are within the reach of the learning subject and can contribute to an effective didactic process for teaching measurement, such as the tape measure, the cell phone and the GeoGebra software. Or even parts of the body, such as a fingernail, according to what was stated by future mathematics teachers.

Affective Suitability, Look at Interests and Motivations

Observation of the interests and motivations that arouse interest in the environment of PMTs leads to reflection in teams on the panorama of problem situations that emerge from the context and facilitate the construction of mathematical meanings during educational practice when they analyze their tastes and semiotic factors that are within their reach (**Figure 6**).

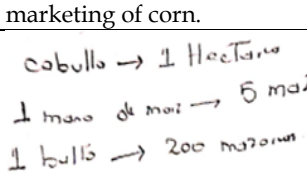
Reflection on the interests and motivations that facilitate the approach to new learning are recognized and used in the planning of problem situations and class material, which encourages dialogue between students on the topic to be taught, as can be seen in the **Table 6**.

The reflection on the interactions between students facilitates the recognition of a dialogue based on the planting, harvesting and marketing of agricultural products, which allows the use of appropriate language during classroom practice, when the topic of measurement is addressed. As pointed out by Cotrado et al. (2022), the promotion of communication and argumentation addresses semiotic conflicts during the construction of mathematical meanings.

Interactional Suitability, Explore Communication Favorable For Learning

Investigating the elements or parts of the body with which people in the studied environment interact at the time of measurement favors the conditions of adequate communication between teacher-students and student-student and guides strategies and didactic material for teaching the topic (**Figure 7**).

Table 6. Recognition of interests and motivations of those who learn

Questions	Answers E2	I1 feedback	Posterior analysis
What type of problem situation, related to measurement, would you like to work on?	Taking advantage of the fact that Suan is an agricultural municipality, we can implement measurement processes in the planting, harvesting and marketing of corn.	Present situations lead you to a connection.	AP301–The management of agricultural scenarios favors learning related to different measurements.
What type of measurement situations do you relate to or have you related to?		The cabuya is a surface measure, which is usually used in this region to measure land areas and corresponds to one hectare. Like the hand of corn, which is 5 ears and the bundle that corresponds to 200 ears.	AP302–The recognition of the cabuya as a surface pattern makes it easier to approach the conventional unit of the hectare.
Make groups of three people and share the answers obtained with your classmates and give an example that for you has contributed to your training as high school or undergraduate students in mathematics.	Title: Rescue of terminology of the traditional metric system. Dialogue is observed in the team, each student gives their opinion and takes notes.	7 groups are organized, and the researcher goes through each of them observing the reflection and conversation carried out, generally repeating the ideas that have arisen in the groups.	AP303–The dialogue on planting, harvesting and marketing of agricultural products, especially corn, generates problematic situations on the issue of measurement.

Note. The image of the relationship between context patterns and measured units corresponds to the answer given to question #2 of group 3 of questions by student E2

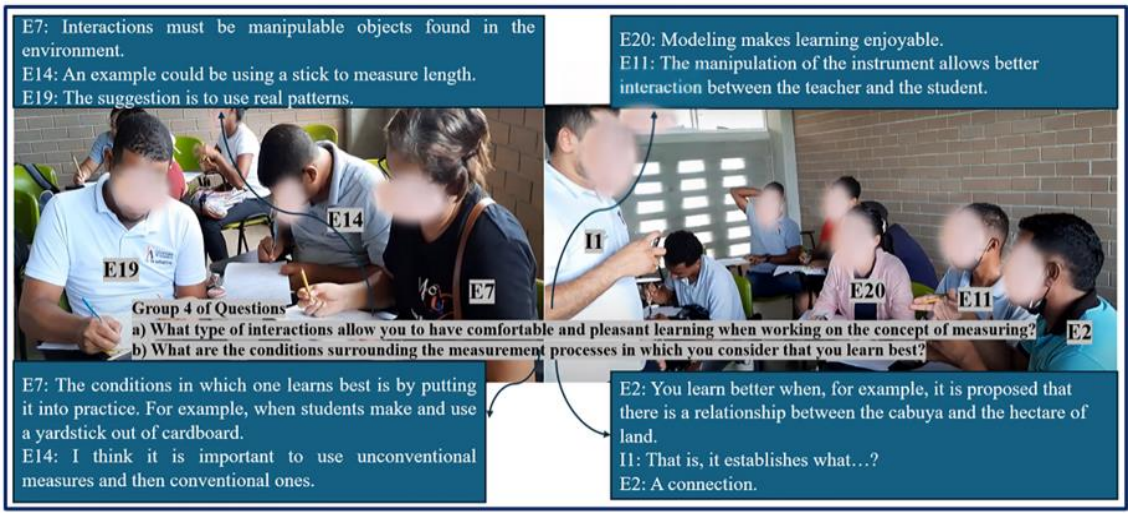


Figure 7. Sharing different experiences about the action of measuring (Source: Authors' own elaboration)

The questions in **Table 7** seek to recognize the interactions and conditions known by PMTs in their environment, then present the feedback from researcher 1 and encourage activity that ignites a conducive dialogue on the topic of the measure.

It has been possible to determine the influence that emotions have on learning, which are influenced according to Ledezma et al. (2021) by the needs, interests, emotions and attitudes of the student. That is why by including in the class the use of unconventional measurements, such as measuring with some parts of the body such as the fingernail, the fourth, the navel, the elbow or using some terms specific to the region such as the cabuya or the quarter of earth, or even evoking memories of a family member taking time with the burning of tobacco and spitting it or selling hands of fish or corn, are significant for the subject who learns and facilitates without discussion the approach more effectively to learning about measurements.

Ecological Suitability, Establishes Curricular Parameters

By looking within the planning process in the pedagogical practices of PMTs at the curricular references to which they refer and articulate with the educational project of the educational institution (**Figure 8**), it is possible to reflect on the connection of the school's mandatory learning and the situations that the subjects who learn about measuring, measurement, unit of measurement, among others, live day to day.

Table 7. Interactions and recommended conditions for teaching measurement in Suan

Questions	Answers E2	I1 feedback	Posterior analysis
What type of interactions allow you to have comfortable and pleasant learning when working on the concept of measuring?	Manipulation and modeling, then I think, that the student learns by manipulating the tools and contexts.	The strategies used during practice arise from what the student knows and likes, to get their attention.	AP401–Manipulation and work with measurements taken with the nail, the hand-span, the navel, the pencil, the yard, the elbow, the rope and the quarter of earth, the well, the fish and corn hand, the totem, the burning of a tobacco, the number of songs and the grandfather's saliva, are meanings that promote dialogue and affection in the investigated context, contributing to the teaching of the measurement topic.
What are the conditions surrounding the measurement processes in which you consider you learn best?	I do lead my students to connect conventional and unconventional measurements.	You how do they learn best? - Manipulating - Practical theory - Towards reality	

Note. The answers in the second column were given by student E2 for group 4 of questions

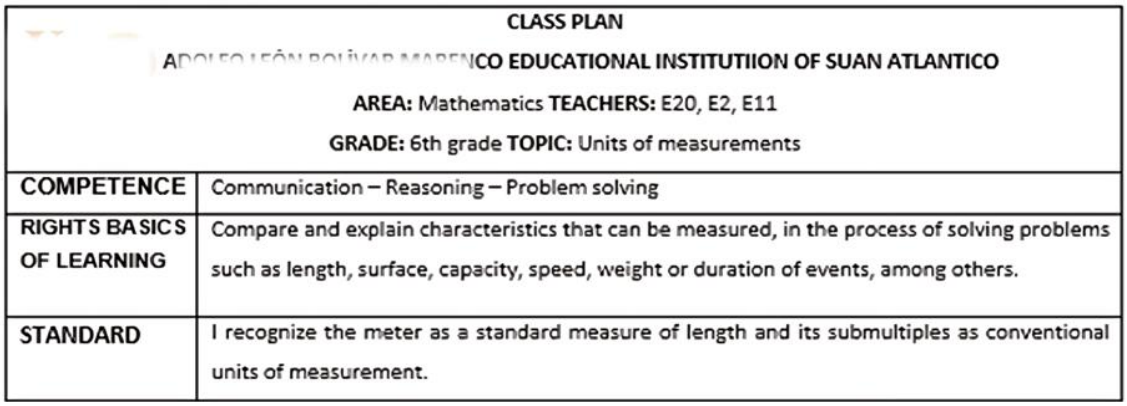


Figure 8. Curricular references used in planning the measure (the PMTs propose an example of the lesson planning for the measure) (Source: Authors' own elaboration)

Table 8. Curricular references, which guide the topic of measurement

Questions	Answers E2	I1 feedback	Posterior analysis
What curriculum elements would you use to address the concept of measurement?	Curriculum reference: I recognize the meter as a standard measurement of length and its submultiples as conventional units of measurement; compare and explain characteristics that can be measured, in the process of solving problems such as length, surface, capacity, speed, weight or duration of events, among others.	During lesson planning, it is necessary to recognize: - The guidelines. - Standards - Basic learning rights. - Curriculum meshes. According to the grade, the quality reference is determined before planning.	AP601–The basic proficiency standard for fifth grade, seeks: 1. The selection of units, both conventional as standardized, appropriate for different measurements. 2. The difference and order, in objects and events, properties or attributes that can be measured. 3. The use and justification of the use of the estimate to solve problems relating to social, economic life and sciences, using ranges of variation

Note. The answers in the second column were given by student E2 for group 6 of questions, teamwork

Group team response to question #6

Next, it is possible to identify in **Table 8**, the curricular references of special and metric thinking for a specific grade, which guide the teaching-learning process for the topic of measurement. These are connected by the researcher, with the purpose of defining institutional meanings and facilitating the understanding of the processes.

For class planning in Colombia, it is necessary to follow quality benchmarks established by the MEN, in order to respond to a spiral curriculum that trains in the development of thinking for problem solving in the different cycles of formation. According to Breda et al. (2018) and Ledezma et al. (2021) to these guidelines, adjustments are made that respond to the institutional educational project, achieving connections between the real context of the student and the guidelines that guarantee the educational quality of the country.

Indicators of Didactic Suitability For Teaching Measurement

Once a follow-up and analysis of the implementation of the task on the topic of the measure has been carried out, researchers and PMTs establish relationships between criteria, components and subsequent analysis, to propose indicators of didactic suitability, which guide pedagogical planning and provide better teaching-learning processes in the lesson attended, as shown in **Figure 9**.

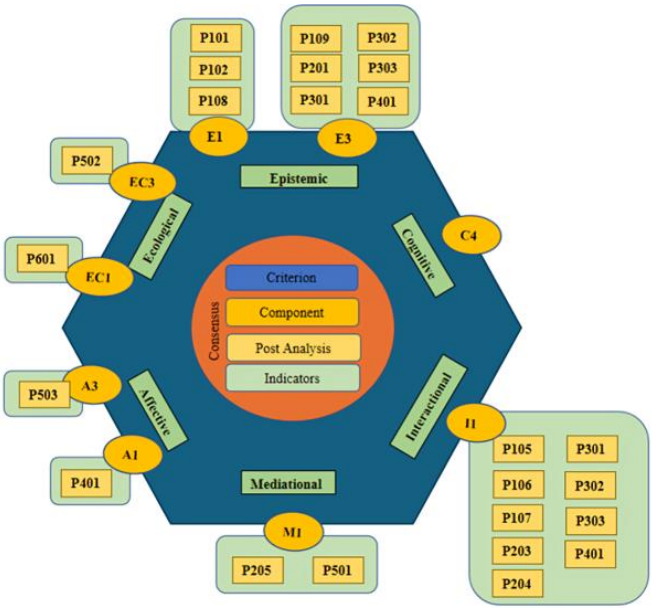


Figure 9. List of criteria and components of didactic suitability (Source: Authors' own elaboration)

In accordance with the reflection that links criteria, components and subsequent analysis, the indicators of didactic suitability that arise from the performance of the measurement task with PMTs are presented below (see **Table 9**).

From **Table 9**, it is possible to see that indicators of didactic suitability emerge, which lead to the following results:

- They make the teacher aware of the need for peer support to recognize errors about the measurement.
- They guide a transdisciplinary curriculum, adapted to the cultural and work usefulness of the environment, on the topic of measurement.
- They strengthen teaching planning processes and contribute to pedagogical practice appropriate to the circumstances that arise.
- Promotes interactions through dialogues that direct the appropriate management of meanings about the measure.
- They favor didactic innovation for teaching measurement, proposing mediations available to everyone.
- They recognize positive emotions, promoting interest in learning about the topic of the measure.

Table 9. List of criteria, components, and subsequent analysis to agree on indicators of didactic suitability

Criterion	C	PA	Didactic suitability indicator for measurement
Epistemic	IE1	AP101, AP102, & AP108	INE1: PMTs: 1. Confuse the concept of measuring with measurement. 2. They limit the meaning of the measurement to that of distance and space. 3. They overlook that a pattern is relative to group agreement.
	IE3	AP109, AP201, AP301, AP302, AP303, & AP401	INE3: They are considered a richness of the process: 1. Starting from patterns, such as the hand of corn and fish, as an introduction to the topic of weight measurement; 2. Use the nail, the hand-span, the navel, the pencil, the yard, the elbow, the rope and the hand-span of land to induce the topics of length and area; 3. Propose problem situations in agricultural scenarios; 4. Establish activities based on the duration of various songs, to teach time measurement.
Interactional	II1	AP105, AP106, AP107, AP203, AP204, AP301, AP302, AP303, & AP401	INI1: Teacher-student communication: 1. It arises from the representation of a measurement from the nail, the fourth, the foot, the rope, the elbow, the height of the navel, the hand of corn, the bowl of rice, the duration of a certain number of songs and the burning of tobacco; 2. Generates dialogues that involve the meaning of conventional measured units of length, weight and time, when reflecting on the planting, harvesting and marketing of agricultural products.
Mediational	IM1	AP205 & AP501	INM1: The use of some resources, such as: the tape measure, the mark on a bowl of rice, the stopwatch and some parts of the body, promote the learning of measurement.
Affective	IA1	AP401	INA1: Taking measurements with the nail, the fourth, the navel, the pencil, the yard, the elbow, the rope and the quarter of earth, the well, the hand of fish and corn, the totem, the burning of a cigar, the number of songs and the grandfather's saliva, awaken interest in the topic of measurement.
	IA3	AP503	INA1: Raising problem situations that arise from interacting on the local court promotes positive emotions that facilitate learning the measure.
Ecological	IEC1	AP601	INEC1: The following purposes are considered to be adopted in the context studied: 1. Develop skills to measure lengths, weight and time, with standardized measurement units, based on usual measurement patterns and make conversions between them; 2. Establish difference, order and price of agricultural products according to the measurement of their attributes; 3. Estimate prices of agricultural products, according to the measurement of length, weight or time.
	IEC3	AP502	INEC3: It is included in the curricular proposal to pose and solve problem situations on the municipality's soccer fields and on the management of agricultural products, which involve measurements of length, weight and time.

Note. C: Component & PA: Posterior analysis

DISCUSSION

The criteria and indicators of didactic suitability obtained as results of this research are the principle of pedagogical relationship to establish best educational practices about measurement. These arose in the search for a teaching-learning process that brings personal meanings closer to institutional meanings, as established by the theoretical foundations based on Godino et al. (2020) and Morales-García et al. (2022) when in the search for training in a mathematical object, the student's previous concepts are analyzed and the social, cultural and political aspects of the context are closely understood.

The discussion on the criteria of didactic suitability in the teaching of measurement, developed in this research with PMTs, highlights the importance of approaching this content from various facets: cognitive, epistemic, affective, mediational, interactional and ecological. These criteria coincide with previous approaches that underline the need for and importance of connecting mathematical content with the sociocultural and emotional context of students (Godino et al., 2003; Godino et al., 2020). The research highlights that the use of non-conventional units of measurement, such as the "nail" or the "elbow", connects the teaching of institutional mathematics with daily practices, a perspective that has also been defended by ethnomathematical approaches (Ávila, 2014; Rodríguez-Nieto, 2021; Rodríguez-Nieto & Alsina, 2022; Sudirman et al., 2024).

From a cognitive suitability perspective, this research investigated how PMTs represent their meanings of concepts, how they teach these concepts, and what problem situations they use to address the issue of measurement. Through this analysis, the didactic approaches used by PMTs are reflected upon and the importance of understanding the particularities of the educational and cultural context in which they teach is highlighted. This approach also emphasizes the need to connect mathematical theory with the daily experience of students, which facilitates more meaningful and contextualized teaching.

Furthermore, research results show that PMTs face several difficulties when teaching measurement. In particular, they tend to confuse basic concepts such as "measuring" and "measurement", limiting the understanding of magnitude and reducing it to distance or space (Sánchez-Matamoros et al., 2018; Sala-Sebastià et al., 2022). This aligns with the findings of other studies, which identify the tendency to apply mechanical procedures, such as the rule of three, without fully understanding the underlying conceptual relationships (Pla-Castells et al., 2021; Mengual et al., 2017). Also, PMTs show difficulties in arguing mathematically in problem solving, indicating the need to promote a deeper and more reflective understanding of measurement concepts (Asil-Güzel & Yeşildere-İmre, 2024).

The present study also highlights the affective and ecological facet, emphasizing the importance of connecting measurement teaching with the local and cultural context of students. Using problems related to everyday life, such as agricultural or sports situations, generates greater interest and facilitates learning for students from different school grades (Rodríguez-Nieto, 2021; Vanegas et al., 2019). This approach reflects the need to design teaching sequences that incorporate both academic content and the sociocultural realities of students, which improves motivation and engagement.

Mediational resources also play a crucial role, for example, the use of concrete materials, such as measuring tapes, clocks and parts of the human body (hand, elbow, etc.), facilitate the learning of measurement. However, oversimplification of procedures should be avoided, which can lead to superficial teaching. The resources must be accompanied by a critical reflection on the mathematical processes involved (Giacomone et al., 2018; Hummes et al., 2023).

Regarding the didactic suitability for measurement, research results suggest that teaching should be transdisciplinary and consider both conceptual aspects and the sociocultural and emotional contexts of students. It is essential that PMTs develop pedagogical skills that allow them to reflect on their practices and connect mathematical concepts with students' everyday experiences (Burgos et al., 2020; Esqué de los Ojos & Breda, 2021). It should be noted that the onto-semiotic vision provides a valuable framework to guide reflection on the teaching of measurement, promoting an integration of mathematical content with cultural and social contexts.

Finally, the research confirms that teaching measurement is a complex challenge that requires teaching methods that combine conceptual understanding, appropriate use of resources, and connection with the sociocultural context, promoting meaningful and reflective learning. Reflection on the representation of meanings and the selection of problem situations is essential to adapt teaching to the needs of students and the particularity of each educational context.

CONCLUSIONS

This research identified the criteria for didactic suitability that emerge in the measurement practices of PMTs, thus fulfilling the proposed objective. These criteria highlight the importance of teaching that not only focuses on abstract mathematical concepts, but also considers the sociocultural, emotional, and epistemological contexts of students. Throughout the study, the relevance of connecting students' personal meanings with the institutional meanings of

mathematics was underlined (Godino et al., 2020). In addition, the use of non-conventional units of measurement was emphasized, such as the "nail" or the "elbow", the "hand-span", which allow linking the teaching of measurement with daily practices.

The research also revealed that PMTs must reflect on their own teaching practice, analyze how they represent mathematical concepts, and consider the specific context in which they teach, which favors more meaningful teaching. The importance in teaching measurement in PMTs is permeated by personal and institutional meanings that, when involved through adequate didactics and treated according to the guidelines of the OSA, favor the construction of the mathematical object. Recognizing the didactic suitability of these subjects, through a questionnaire-type workshop that was designed according to one of the analysis tools of that theory of mathematics didactics, applied during classroom practice in the development of a task and analyzed by researchers under the structure and feedback based on the theory of didactic suitability.

The analysis of this instrument made it possible to establish a relationship between the responses to the task of PMTs, the researcher's feedback and a subsequent analysis, guided by agreed criteria and components, to reflect the importance in educational research of analyzing the social and cultural context when better learning is intended. Reflection on the cognitive, epistemic, interactional, mediational, affective and ecological, leads to establishing suitable indicators that guide teaching about measurement, based on patterns common to an area or place, to better understand the standardized units of measurements.

The recognition of the language used in the interaction of practice when solving a measurement task, the evocation of family, social and cultural experiences of the topic, curricular reflection and dialogue according to the tendency to make errors and take advantage of richness of processes, allow the construction of didactics, based on problem situations that give meaning to learning. Finally, these results are useful for future research where in-service teachers apply workshops with their preschool, primary and secondary school students where metric and spatial thinking is promoted in a way connected to the sociocultural context of the students.

Author contributions: MPG-M: conceptualization, methodology, supervision, and writing–original draft & **CAR-N, APA, & AB:** formal analysis, validation, writing–original draft, writing–review & editing, and resources. All authors agreed with the results and conclusions.

Funding: This study is part of the projects proyecto de docencia codificado por DOC.100-11-001-18 (Universidad de la Costa) & grant PID2021-127104NB-I00 funded by MICIU/AEI/10.13039/501100011033 and by "ERDF: A way of making Europe".

Ethical statement: The authors stated that the study does not require any ethical approval because the first two authors of this article are professors in charge of the research subjects that the students were taking at the aforementioned university and have sufficient permission to work with the students on this type of educational projects and teach them theoretical and methodological perspectives of Mathematics Education. Written informed consents were obtained from the participants.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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