MODESTUM EURASIA Journal of Mathematics, Science and Technology Education, 2024, 20(12), em2548
ISSN:1305-8223 (online)
OPEN ACCESS Research Paper https://doi.org/10.29333/ejmste/15704

A socio-epistemological approach articulated with problem-solving in higher education: Teaching of integral calculus

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Received 18 May 2024 ▪ Accepted 05 November 2024

Abstract

The training of mathematics teachers in universities in Colombia has as a transversal axis the resolution of problems based on their social and cognitive mission pillars. In this sense, this study relates to the re-signification and construction of the concept of definite integral (DI) (integral calculus) through socio-epistemological studies, action researches, and the typology of didactic situations. The results are obtained through content analysis, didactic sequences (GeoGebra), and a discussion group. The above allows us to conclude that the validation of meanings, historical contexts, and associated social practices leads to the construction of the concept of DI as a model of mathematical analysis. This structuring of knowledge from its epistemological framework enables the exploration of mathematical objects from the basic notions that emerge in the history of humanity and didactic processes that reconstruct the evolution of the concept in society.

Keywords: higher education, socio-epistemological research, GeoGebra®, teacher training, problem-solving

INTRODUCTION

In the development of educational praxis and specifically for the teaching of mathematics, class planning and design is configured as a fundamental axis in which teachers apply didactic transposition methods (Chevallard, 1991). In this sense, the process of transforming knowledge into a teaching object emerges from communities of practice and personal knowledge. Godino et al. (2013) associated with beliefs and conceptions about problem-solving (PS) as content, final application of a mathematical object, as a purpose of teaching and learning, or the purpose of use, sense and meaning obtained by mathematical concepts in social and cultural contexts that are situated within the dynamics of human activities (López-Leyton et al., 2018).

Therefore, in the teaching of the fundamental concepts of integral calculus (IC), the emphasis remains on solving repetitive exercises, without involving PS in real-world contexts or situations related to cultural, historical, or professional knowledge. As a result, it is necessary for researchers to study and create spaces and dynamics that contribute to improving these practices (Serrano, 2010). For this reason, instrumental teaching and the lack of a real context create epistemological gaps in students, which become evident when they attempt to tackle real-life or everyday situations. This is due to the absence of guidance in investigating and recognizing patterns, formulating conjectures, appropriating different representation systems, establishing connections, utilizing variety in the argumentative process, and communicating their findings (Santos & Vargas, 2003).

According to the challenges identified in the literature and professional experience regarding the comprehension and social construction of the concept of 'definite integral (DI),' it is asserted that there is a need to analyze the following inquiry from a research perspective: How does a socio-epistemological approach, integrated with PS in context, enable a re-signification of the construction of the DI concept in the training of prospective mathematics teachers in the

This article is the result of the research work titled "A socio-epistemological approach of the definite integral from problem-solving as learning strategy" in the master's program in education science at Universidad del Quindío.

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Contribution to the literature

- This article highlights the importance of integrating critical thinking into mathematics education.
- It provides a theoretical and methodological foundation to strengthen PS skills and the understanding of abstract mathematical objects, thereby facilitating real-world applications from a social and cognitive perspective.
- It also equips teachers with tools to adopt more effective educational practices aligned with current demands, promoting deep and relevant learning among students.

mathematics education program at the University of Quindío? To this end, the general purpose of this research is to re-signify the construction of the DI concept through a socio-epistemological approach and PS as a learning strategy in the training of mathematics teacher candidates during the established curricular period within the IC course.

THEORETICAL FRAMEWORK

The theoretical aspects underlying this research are advanced mathematical thinking (AMT), socio-epistemology, and PS as learning strategy; for this purpose, the socio-epistemological theory was established as the lens that permits visualizing the constitutive and situational aspects from which the concept of DI emerges. Thereby, it becomes necessary to resort to the problematization of knowledge as a strategy to stimulate processes necessary to develop AMT.

Advanced Mathematical Thinking

According with Azcárate Gimenez and Camacho-Machín (2003), the dissertation on AMT has its origins within the framework of the 1985 Congress on the Psychology of Mathematics Education, when a working group was comprised between Gontran Ervynck and David Tall regarding research on teaching-learning processes related with infinitesimal calculation, result of the analysis of mathematical thinking to be developed during the last years of high school and university courses, which consider definition, test, and demonstration processes by students. Consequently, the research line "didactics of the mathematical analysis" emerges to respond about the principal characteristics of AMT (Aldana et al., 2020). In this respect, Tall and Vinner (1981) indicate that the definition of a concept linked to AMT is a succession of words or verbal representations product of their historical progress according with the Definition concept – Image concept. From this structure emerge different theoretical aspects, such as the stages or moments in the progressive structuring of a concept (interiorization-condensation-reification) (Sfard, 1991) and the dialectic of the 'procept' (Gray & Tall, 1994), where the symbol forms a meeting point between process and object. These are essential theories for the development of this study.

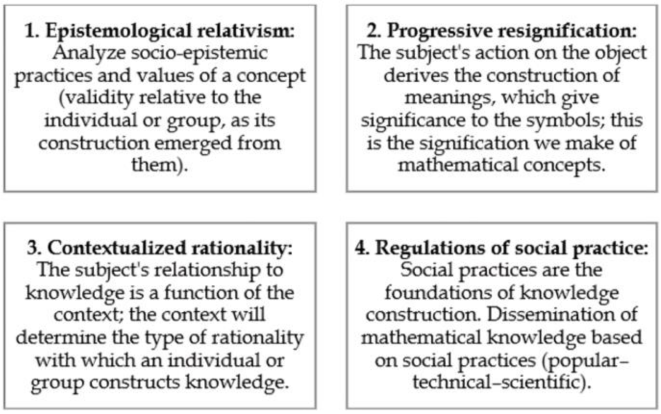

Figure 1. The socio-epistemology theory (Cantoral, 2011)

The Socio-Epistemological Theory of Educational Mathematics and Problem-Solving as Learning Strategy

The socio-epistemology theory, according to Cantoral et al. (2014), assumes as fundamental idea that to study didactic phenomena linked to mathematics it is necessary to undergo a thorough examination of knowledge, to its problematization (**Figure 1**). Socio-epistemology makes an important contribution: it models the dynamics of knowledge or "knowledge put to use". To achieve this, it was indispensable to introduce the notion of use, in contrast with the psychological notion of acquisition through learning; going from static knowledge to the study of knowledge in use, that is, to the study of knowledge. It is essential to clarify that within this approach, all forms of thought, whether popular, technical, or scientific, are considered legitimate, as together they constitute human knowledge.

Socio-epistemology is based on four fundamental pillars forming a nodal network: the principle of contextualized rationality (Cantoral et al., 2014). This principle alludes to the fact that the relationship between subject and knowledge is a function of the context that is indispensable for understanding the normative principles of reasoning within the specific contexts under which an inference is made. The principle of epistemological relativism. Relativism is the concept that holds that points of view have no universal truth or validity, but, in any case, they only have a subjective and relative validity to the different frames of reference. The principle of progressive resignification focuses on genetic epistemology in which action is the base of the development of knowledge, the action of the subject over the object, therein derive the constructed meanings. Hence, significance will depend to a greater extent on the contextual scenario where it is produced. The normative principle of social practice is the fundamental link for the theory to function. It is assumed that social practices are the base and guide in knowledge construction processes, constituted as generators of knowledge.

According with Gómez (2005), problem-based learning "is a didactic method, which falls on the domain of active pedagogies and, particularly, on the teaching strategy called learning by discovery and construction, opposed to the masterful exhibition strategy" (p. 10). In this sense, the students are the center of their own process; they search for information, select that which is necessary, organize and propose strategies, guess, and apply processes to solve the situations the teacher proposes. This is why the teacher's role in this methodology is that of "a guide, an expositor of problems or problematic situations, who suggests sources of information and is ready to collaborate with the learners' needs" (Gómez, 2005).

The theoretical aspects mentioned permit comprehending the epistemological scenario associated with the didactics of AMT, making it necessary to have a methodological framework that allows putting these approaches into practice, as shown in the following section.

METHODOLOGY

This critical-social research (Jiménez & Cerón, 1992) sought the articulation between theory and observation, between context of discovery and justification, that is, it assumes that knowledge depends on its social framework as on its internal structure. According with Bisquerra (2009), it is a qualitative research aimed at comprehension and change (conceptually framed in critical theory), thereby, proposing the shared need to transform methods and consider the trajectories a learner follows when seeking to understand any object of knowledge (López-Leyton et al., 2019).

This study kept in mind action research (AR) (Latorre, 2007) focused on the teachers' performance as determining factor for change, transformation, and empowerment of the pedagogical preparation. The following describes the phases of the AR process according with Bisquerra (2009):

1. **Planning:** This has to do with the description and interpretation of the research problem and the design of the strategic action.
2. **Action:** It refers to the implementation of strategic action, promoting the careful and reflexive change of professional practice. It is a deliberate and controlled process that permits systematic data generation to collect the rationale that supports the evidence about practice.
3. **Observation:** In this stage, the genesis of the data is monitored to reflect, evaluate, and explain what happened.
4. **Reflection:** This constitutes the final stage of the cycle that allows identifying meanings, theories, and possible models to reframe the problem.

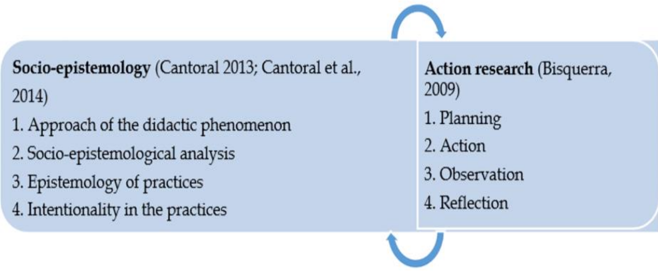

Figure 2. Articulation socio-epistemology and artificial intelligence.

From the aforementioned, an adaptation is established of the four moments of AR to the four phases of socio-epistemological research, as shown in **Figure 2**.

The setting for this research was at Universidad del Quindío (Colombia), in the mathematics degree program with 11 students from the IC course. This academic space (formation variable in calculus II) is offered during the fourth semester with an hourly intensity of 4 hours per week, for a 16-week period with a total of 64 hours.

Information Collection Techniques

For the classroom intervention, within the framework of this research, two data-collection techniques were considered, like:

1. **Questionnaires:** This instrument, called didactic learning sequence (Camargo et al., 2008), is a set of activities organized, structured and articulated around a central topic that seeks to analyze how students use the mathematical elements that constitute Riemann's DI concept in PS or mathematical tasks in a socio-epistemological context.
2. **Discussion group:** According to Bisquerra (2009), it is constituted as a qualitative technique that resorts to the interview conducted with a whole group of people to gather relevant information on the impact of the intervention.

ANALYSIS AND DISCUSSION OF RESULTS

This research began with three types of analyses (planning):

1. **Analysis of the sociogenesis of the concept:** This observation process allows taking a historical

Table 1. Learning trajectory of the DI concept in this research

Pillar (socio-epistemology)	Sequence and activity	Founding ideas to construct the DI concept
Principle of epistemological relativism	Sequence 1 Activity 1 Situation in context: Study of Zeno's paradoxes	- Intuitive idea of approximation and limit - Dynamic conception of limit (infinite of the uncountable or divergent): Subdivisions in a number of infinite or uncountable parts - Metric conception of limit (infinitely small or convergent): Distances so small that they tend to zero - Possibility of dividing a finite distance into an infinite number of sections - Existence of real infinite numbers in numerical intervals - The sigma notation as mathematical tool to represent sums - Properties of the sigma notation.
	Sequence 1 Activity 2 Situation in context: The exhaustion method as part of the study of eclipses	- The circumference as geometric figure with infinite sides (dynamic conception) whose length tends to be zero (metric conception) - The exhaustion method and its relationship with the area from the practices that originally gave it sense - Construction of the formula for the area of a circumference through analytic methods - The formula of the length of a circumference as that derived from the formula of the area of a circumference.
Principle of progressive resignification	Sequence 1 Activity 3 Situation in context: Approximate land area of a tourist farm	- Curved figures can be decomposed into square figures (rectangles) - Mathematical expressions in terms of summations as an approximation of the area of irregular figures - If the sum of areas of a number of subdivisions of a plane tends to infinite, then the margin of error of the total area tends to be zero.
	Sequence 1 Activity 4 Situation in context: Exact land area of a tourist farm	- Making a greater subdivision of an irregular figure improves the approach to find its area - Use of the limit concept associated with a Riemann sum - The limit of a sum as the exact area resorting to the dynamic and metric conception of infinity. - The DI as the limit of a Riemann sum.
Principle of contextualized rationality	Sequence 2 Activity 1 Situation in context: Growth of a bush sold in a nursery	- The DI as area of a region - La rate of change in function of the area under the curve. - The DI as method of accumulation of differentials - Properties of the DI and its application in solving integrals.
	Sequence 2 Activity 2 Situation in context: Area of a function with negative and non-negative sections	- The DI as the difference between the area over the abscissa and the area under the abscissa" - The DI always yields as result a number from the set of real numbers. - The absolute value of an integral, like the total area between a function (negative and non-negative) and the axis of abscissas - The DI as a number that represents the area of a plane
Normative principle of the social practice	Sequence 3 Activity 1 Situation in context: Lifetime of a person infected with bacteria	- The DI as accumulation function under a rate of change - Fundamental theorem of calculus part one (bridge–or relationship–between DI and IC) - Anti-derivation as an inverse process to derivation - The InDI as a family of functions
	Sequence 3 Activity 2 Situation in context: Lifetime of a person infected with bacteria	- Fundamental theorem of calculus second part (given an accumulation function to find its variation from a to b)

journey from the idea or need that gave rise to the DI concept, until its moment of formalization in the academic community.

2. **Epistemological analysis of the concept's mathematics:** After knowing the genesis and development of the concept, this type of analysis aims to undertake a formal decomposition of the elements that constitute the mathematical object of study.
3. **Analysis of meanings and contexts acquired by the concept:** This has to do with the multi-disciplinarity and interdisciplinarity reached by the DI from other sciences. From the aforementioned, the design is configured of each teaching sequence, for the intervention according with the pillars of socio-epistemology, as shown in **Table 1**.

The results are presented in function of the action and observation phase (epistemology of the practices) conducted during the first semester of 2019; due to the foregoing, we have, as source of information:

1. *Some class episodes conducted (**Table 1**) during a course of IC in the mathematics degree program at Universidad del Quindío* and

Figure 3. La Paloma farm activity (Source: Screenshot from Google Maps)

2. *A discussion group with the students who participated in the research.*

The record of the intervention was obtained from learning sequences configured by the approaches and procedures of the students, photographs, and transcription. The following presents the information collected from some of the learning sequences in light of the socio-epistemological approach:

Class Episodes

Principle of progressive resignification

Didactic sequence 1 (S1)–Activity 3 (A3): To carry out this activity, the teacher during the action phase asked to form couples and delivered a learning sequence in which the students had to solve a problem situation. This problem situation designed by the teacher addresses a context typical of the department of Quindío, such as tourism farms. The following presents the situation posed.

Mr. Tito Leyton Villamil owns La Paloma farm in the Bohemia rural area. Currently, he is carrying out the procedures to register and legalize his land as a tourist farm in the local Chamber of Commerce. To fill out these documents, the property owner must approximate the area of his land. Mr. Tito does not have enough money to hire a surveyor, thus, he has asked Carlos, a friend who is a mathematician, to help him. Carlos used Google Maps (**Figure 3**) to see the location of the land and calculate its area, therein, he found that the farm borders another property and the rural roadway, as shown in the following image. With you being a mathematician who will help Mr. Tito, how would you find the land area?

The following are some answers by the students and their tendencies.

In **Figure 4**, student E1 responds, "the area is approximately 250 km²." Student E2 determines that "the area of the triangle" "approximates the area of the farm; a more accurate method would be to inscribe shapes with more sides by dividing the figure in half, thus getting closer each time."

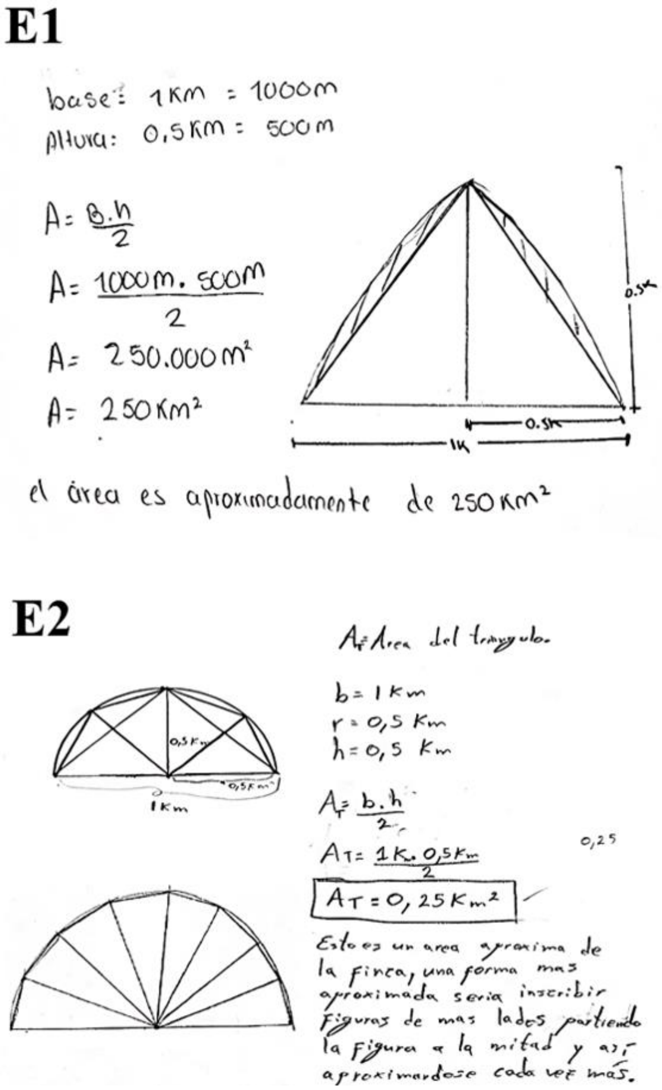

Figure 4. Student production-1 (Source: Field study)

Tendency: Until now, the students were under the expectation of what was addressed during each session. Work in this action phase was more comfortable for them because with the ideas from previous sessions and their skills in executing processes, they were able to establish solutions without fear of being wrong. Student **E1** (**Figure 4**) based the solution on the analysis of prior classes on the exhaustion method as tool to find an approximation of the land area.

Student **E2** (**Figure 4**) also resorted to the exhaustion tool. But, upon realizing that only one approximation was found, the student proposed–based on the arguments from previous sessions–a process to find approximations increasingly closer to the real land area.

After providing a period of time, the teacher asked the students to hand in their processes. Then, another learning sequence was given with some questions, which were collected by the teacher prior to the discussion:

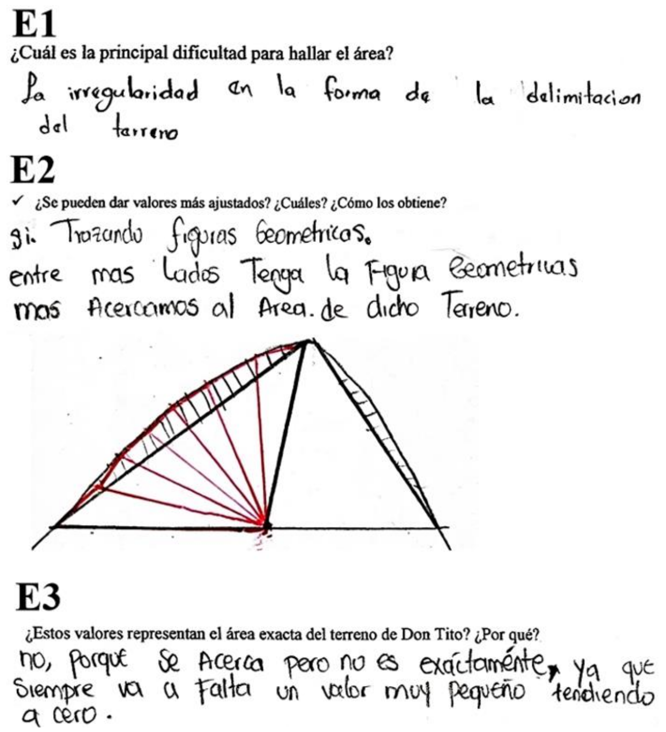

Figure 5. Student production-2 (Source: Field study)

In **Figure 5**, **E1** is asked, What is the main difficulty in finding the area? to which they respond, "The irregularity in the shape of the land's boundaries." For **E2**, the question is, Can more accurate values be provided? Which ones? How are they obtained? The response is, "Yes, by drawing geometric figures; the more sides the geometric figure has, the closer we get to the area of that land". **E3** is asked, do these values represent the exact area of Don Tito's land? Why? The response is, "No, because it gets closer but is not exact, as there will always be a very small value approaching zero that is missing".

Tendency: During this activity, the students had the opportunity to argue about their processes and conclusions. Students **E1** (**Figure 5**) and **E2** (**Figure 5**) recognize the difficulty of finding the exact area of the land and use a deductive approach from previous classes on exhaustion to propose a solution that can provide a better approximation of the area. This leads to the conclusion that, up to this point, the students are in a process of internalizing condensation (Sfard, 1991), as they recognize the intuitive idea from which the calculation of areas arises and attempt to apply it in a real situation. Student **E3** (**Figure 5**) acknowledges the difficulty of only finding approximations where the margin of error begins tending to zero.

From the activity and the previous discussion, the teacher hands out the problem situation in print for the students to have in their notebooks and institutionalized:

A possible solution: Considering the thoughts of Aristotle and Archimedes, in whose principle a curved figure can be decomposed into square figures (quadrature of the circle), we will approximate the land area by decomposing it into n rectangles of equal base but using as tool the curve on the road to make a calculation. In this sense, we must find the equation of the parabola that models the property's roadway. For such, we will locate a cartesian plane in which the property is in the first quadrant, as shown in **Figure 6**. Then, geometrically, it is possible to represent the situation with the following graphic:

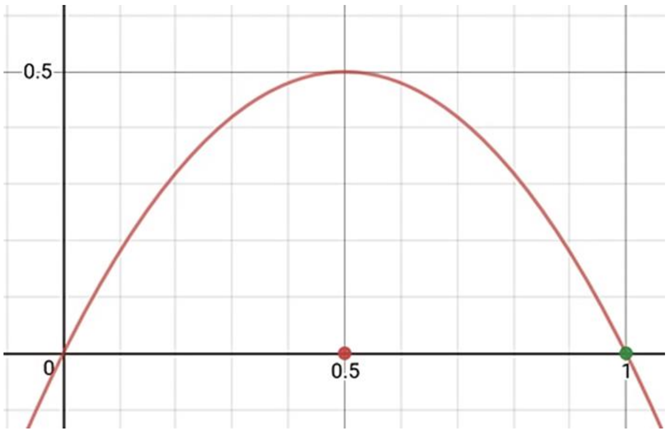

Figure 6. La Paloma farm activity solution (Source: Authors' own elaboration)

In this sense, it is possible to visualize that points $x = 0$ and $x = 1$ are the cuts that the graph presents with the x-axis. From the course on differential calculus, we know that those points represent the roots of the function. Then, the correspondence rule of the function is given by the expression:

$$y = a(x - 0)(x - 1). \quad (1)$$

Now, take any point to find the value of a; we will take point $(0.5, 0.5)$ or $\left(\frac{1}{2}, \frac{1}{2}\right)$ through which the curve passes. This point is facilitated by the graphic, given the land measurements. Replacing in Eq. (1), we have:

$$\frac{1}{2} = a\left(\frac{1}{2} - 0\right)\left(\frac{1}{2} - 1\right) = a\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) = a\left(-\frac{1}{4}\right),$$
$$\frac{\frac{1}{2}}{-\frac{1}{4}} = a, \quad (2)$$
$$a = -\frac{4}{2} = -2.$$

Hence, the correspondence rule that models the roadway's behavior is:

$$y = -2(x)(x - 1) = -2(x^2 - x) = -2x^2 + 2x = 2x - 2x^2. \quad (3)$$

Then, the problem is reduced to finding an expression that approximates the area under the curve $f(x) = 2x - 2x^2$ in the interval $[0, 1]$, which must approach the general formula for n rectangles. At this moment, the teacher proposes the following activity with the GeoGebra software.

Elaboration of sums of rectangles under the curve $f(x) = 2x - x^2$: The teacher tells the students to use an

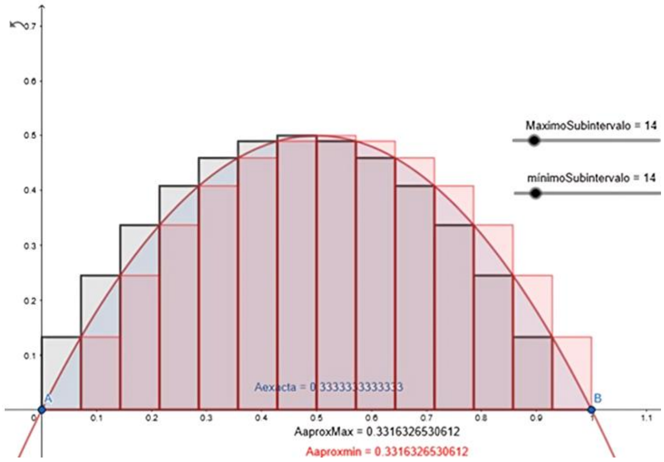

Figure 7. Representation of GeoGebra intervals (Source: Authors' own elaboration)

Android type device or computer to perform an exercise in the GeoGebra software, this activity seeks to improve visualizing the approximations made through sums:

1. Start the GeoGebra application in your device
2. *Representation of the function:* Enter in the writing pad $f(x) = 2x - 2x^2$ and press enter.
3. *Cuts and interval in the x-axis:* Enter in the writing pad $A = (0,0)$ and press enter. Enter in the writing pad $B = (1,0)$ and press enter.
4. *Create slider:* Go to the tools panel and select a slider, name it $minimumSubinterval$, minimum 0 maximum 200, with increase of one unit.
5. *Create rectangles from the leftmost or interval minimum:* Enter in the writing pad $RectangleSum(f, x(A), x(B), minimumSubinterval, 0)$ and press enter.
6. *Move the slider in the screen and observe.*
7. *Create another slider:* Go to the tools panel and select a slider, name it $MaximumSubinterval$, minimum 0 maximum 200, with increase of one unit.
8. *Create rectangles from the leftmost or interval minimum:* Enter in the writing pad $RectangleSum(f, x(A), x(B), MaximumSubinterval, 1)$ *and press enter.*
9. *Move the slider in the screen and observe:* Go to the main menu and save the file in the cloud or download it to your device.

Now, the teacher uses the GeoGebra software (**Figure 7**) to illustrate to students both types of approaches that have been carried out.

The problem is reduced, then, to finding an expression (Riemann sum) that allows giving values to n (number of partitions or rectangles) and, thus, have approximations of the real land area with the conditions given; then, the teacher illustrates how to find an expression with n subintervals in the partition of this area, when obtaining as result that:

Table 2. Activity S1–A4

Number of partitions n	Approximation under the expression $A \approx \left[\frac{(n+1)(n-1)}{3n^2}\right]$
$n = 4$	
$n = 10$	
$n = 20$	
$n = 40$	
$n = 50$	
$n = 100$	
$n = 200$	
Another? $n =$ ________	

$$A \approx \sum_{i=1}^{n} f(x_{i-1})\,(\Delta x) = \sum_{i=1}^{n}\left[2\left(\frac{i-1}{n}\right) - 2\left(\frac{i-1}{n}\right)^2\right]\left(\frac{1}{n}\right) = \frac{(n+1)(n-1)}{3n^2}. \quad (4)$$

With $n = number\ of\ partitions\ or\ rectangles$. It is a general formula (Riemann sum) to approximate the land area.

Didactic sequence 1 (S1)–Activity 4 (A4): To conduct this activity, the teacher gave students a learning sequence linked with the previous. In it, the students must fill out a series of approximations, given the mathematical expression found in the last class; from there, the mathematical concept foreseen in the planning emerges. The activity consisted of in.

Carlos, Mr. Tito's mathematician friend, has reached the conclusion that expression $\frac{(n+1)(n-1)}{3n^2}$, where n is the number of land subdivisions, models an approximation of the property's area. However, he wishes to provide an exact number to better fill out the documents. Hence, he constructed **Table 2** to see the behavior when using increasingly more subdivisions n. Complete **Table 2** for the approximations of Mr. Tito's land area:

1. What can you conclude about the dynamics proposed by Mr. Tito's friend on **Table 2**?
2. Would it be possible to calculate the exact land area? Which one? How is it obtained?

The following presents some answers by the students and their tendency.

In **Figure 8**, **E2** is asked, "What can you conclude about the approach suggested by Don Tito's friend in the table?" to which he responds, "The larger the partition, the better the approximation of the area." For **E3**, the question is, "Would it be possible to calculate the exact area of the land? Which one? How would you obtain it?" The response is, "Yes, by approximating n to infinity".

Tendency: Up to this point, the students are clear that making a greater subdivision of the area improves the approximation and also that it is possible to find expressions that model through a variable n as the number of sides, approximations that tend to the exact area of the figure, as evidenced by the processes and arguments by students **E1** (**Figure 8**) and **E2** (**Figure 8**).

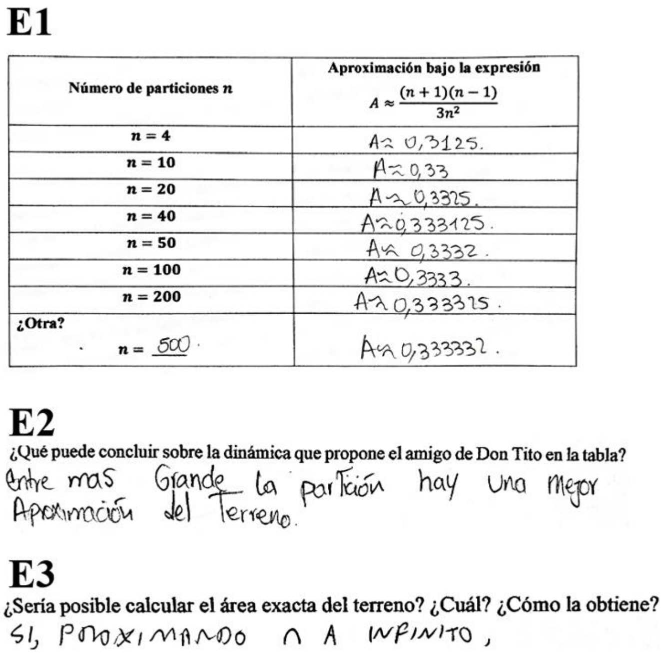

Figure 8. Student production-3 (Source: Field study)

For student **E3** (**Figure 8**), it can be observed that the student understands that, through Riemann sum processes, it is not possible to find the exact area, as it could only be found when the number of intervals tends to infinity ($n \to \infty$) and, thereby, it is necessary to use the concept of limit.

Based on the previous activity and the collective debate, the teacher intervenes to initiate the validation process by presenting some of the students' processes on the classroom board and discussing them to achieve the following process:

Given that we have the expression that approximates area $A \approx \frac{(n+1)(n-1)}{3n^2}$ Would it be possible to calculate the exact land area? Yes, of course. We must, then, consider that $n \to \infty$, that is, a larger n means better approximation to the exact land area. Therefore, we have:

$$A = \lim_{n\to\infty} \frac{(n+1)(n-1)}{3n^2} = \lim_{n\to\infty} \frac{n^2-1}{3n^2} = \lim_{n\to\infty} \frac{n^2}{3n^2} - \frac{1}{3n^2} = \lim_{n\to\infty} \frac{1}{3} - \frac{1}{3n^2} = \lim_{n\to\infty} \frac{1}{3} - \lim_{n\to\infty} \frac{1}{3n^2} = \frac{1}{3} - \left[\frac{1}{3}\left(\lim_{n\to\infty} \frac{1}{n^2}\right)\right] = \frac{1}{3} - \left[\frac{1}{3}(0)\right] = \frac{1}{3}. \quad (5)$$

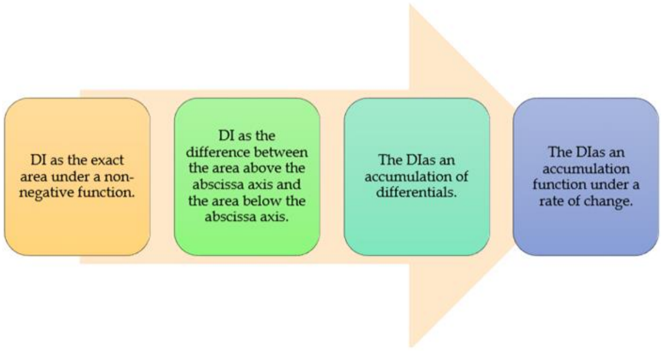

Figure 9. Meanings acquired by the DI (Source: Authors' own elaboration)

Answer: Mr. Tito's exact land area is $\frac{1}{3} km^2$. That is $333.3 \ldots m^2$

The foregoing permitted institutionalizing the concept of **the area as a limit of a sum**.

The meanings (**Figure 9**) and hierarchy of the concept of DI developed in this research are shown below:

From the field work, some categories arise that reveal the different characteristics proposed by the design of didactic situations, through a socio-epistemological approach articulated with PS and the theory of didactic situations as intervention model from AR (**Table 3**):

Through these categories, the script was drawn for the discussion group and the semi-structured interviews. The following presents some fragments of the results about the information collected through these two instruments.

Information Collected From a Discussion Group With the Students Who Participated in the Research

For the last field work session, a discussion group was agreed with the students, which analyzed aspects related to the categories already exposed. The following presents the analysis of the information obtained from the discussion group (reflections by students provoked by discussion questions) in function of the categories emerging from the intervention process (**Table 4**).

The articulation between teaching and research in this study allows for a re-signification of the construction and understanding of the defined integral concept in

Table 3. Categories emerging from the field work

Category	Description
1. Perceptions about teacher training	This has to do with those ideas, opinions, critiques, and arguments of students as to how they sense their teaching formation has been.
2. Perceptions about the didactics of mathematics	This has to do with those ideas, opinions, critiques, and arguments of students regarding the strategies, contents, and methods to design and conduct a class.
3. Self-critical analysis of the teaching practice within a context developed by its own protagonists	This has to do with those ideas, opinions, critiques, and arguments of students regarding their own formation in function of the teaching practices they use in the present.
4. Awareness about the possibilities of change and improvement of the educational practice	This has to do with those ideas, opinions, critiques, and arguments of students regarding the reflection about the teaching profession.

Table 4. Analysis of the students' reflections (discussion group)

Category	Descriptors
1. Perceptions about the teacher training	- The process for good planning and mathematics class is in: 1. Analysis of prior knowledge, 2. Guided discovery process, and 3. Institutionalization of concepts. - Need for teachers to use inductive methods through questions, not definitions - Lack of example-based teaching - Lack of teacher training from the very teaching - Disarticulation between mathematical knowledge and didactic knowledge of the content
2. Perceptions about the didactics of mathematics	- A guided construction of the DI concept through intuitive ideas is necessary. - At university level, a concept (DI) is known when its origin is known, epistemologically located, applied to real contexts, and formally demonstrated. - The biggest difficulty in learning a concept is when it integrates prior knowledge.
3. Self-critical analysis of the teaching practice within a context developed by its own protagonists	- It is necessary for teachers to teach to create mathematics, not only to write them and read them. - From the current teacher training, it is necessary to highlight the acquisition of theories, methodologies, and strategies for the future profession.
4. Awareness about possibilities of change and improvement of the educational practice	- It is necessary to dimension problem solving as a central strategy to teach mathematics. - The development of thought must prevail against the procedural and formal nature of mathematics in a classroom.

students for mathematics teachers, as evidenced by the results. This contributes to the difficulties encountered in studies such as Serrano (2010) and Santos and Vargas (2003). The scope of this classroom intervention from a socio-epistemological approach admitted:

1. The transfer of knowledge through problem-based teaching derived from social practices, everyday life, and other sciences and
2. The initial training of teachers is aware of the need to transform their didactic transposition processes, integrating and systematizing mathematical knowledge within the dynamics of human activities.

Therefore, the socio-epistemological approach developed in this research enables the teacher-researcher to identify epistemological, didactic, and cognitive elements related to the defined integral concept through three types of analysis:

1. Analysis of the sociogenesis of the concept,
2. Epistemological analysis of the mathematics of the concept, and
3. Analysis of the meanings and contexts acquired by the concept.

These three types of analysis allow teachers to acquire all the necessary elements to design class sequences that integrate these aspects and, in this way, create the tools as a response to the need to formulate and execute contextualized activities from which the concepts of mathematical knowledge emerge epistemologically. As a result, the following was obtained:

1. Responding from academic and scientific positions to questions posed by Cantoral et al. (2014): What to teach? Why teach it? For what purpose? How to teach it? When to teach it?
2. Contributing multiple meanings to the defined integral concept and the notions surrounding it.
3. Reconstructing and simulating in the classroom those situations from the historical moment in which problems were posed and the conditions from which the defined integral concept emerges.
4. Drawing on historical research as a source and tool for understanding, as evidenced in the production of participating students.
5. Gaining an expanded view of the defined integral concept, based on the trajectory of social practices that gave it significance from other disciplines, as proposed by Camargo et al. (2008).
6. Establishing models of study for calculus that integrate the development of infinitesimal thinking as a fundamental idea of mathematical analysis.

CONCLUSIONS

Through student participation for the professor within this research process, students were able to establish critical ideas on how they have been learning mathematics and the strategies they will use for teaching; this fosters a desire to find an identity regarding the possibilities for change and improvement in the teaching profession, in alignment with what Aldana et al. (2020) assert. From another perspective, participation in a process designed and centered around social reference practices, PS, and types of didactic situations present in the classroom (Cantoral et al., 2014) leads the involved students to reflect on the need to transform the preparation and didactic transposition processes focused on organizing methods, content, and strategies that seek to reorganize mathematical discourse around sociocultural aspects, the student's environment, and the purpose of learning.

Based on the pillars of the socio-epistemological approach (Gómez, 2005), some epistemological contributions to the teaching of mathematical analysis emerge from this research that validates the architecture of the learning obtained by students on the concept of the DI:

1. **Principle of epistemological relativism:** In this research, teaching fundamental mathematical objects of calculus (derivative and integral) based on infinitesimal thinking as the constitutive genesis "dividing a finite quantity by an infinite number of subdivisions" allows the concept of limit to emerge from its two conceptions (dynamic and metric), facilitating students' understanding of the relationship between differential and IC from the social practices requiring this knowledge (Cantoral, 2013).
2. **Principle of progressive resignification:** This study concludes that, in the processes of didactic transposition, a mathematical problem should be identified as the vehicle connecting the destinations of the acquired and forthcoming learnings, as solving this problem leads to the emergence of a concept needed by the students to resolve it. In this way, the concept does not simply become another title in students' minds but a need arising from their daily lives, as asserted by López-Leyton et al. (2019). Consequently, problematic situations from other sciences (biology and microbiology in this research) allow students to explore knowledge from everyday practical use.
3. **Principle of contextualized rationality:** This study establishes a trajectory for learning the concept of DI. This trajectory articulates intuitive ideas with constitutive mathematical objects associated with the genetic decomposition of this object of study (Aldana, 2011), thus enabling the student to acquire the essential reasoning needed to construct the concept of the DI from scientific rigor and their personal ideas, such as:
   a. Dynamic conception of limit (infinite of uncountable or divergent): Subdivisions into an infinite or uncountable number of parts.
   b. Metric conception of the limit (infinite of the uncountable or convergent): Distances so small that they approach zero.
   c. The possibility of dividing a finite distance into an infinite number of segments.
   d. The existence of infinite real numbers within numerical intervals.
   e. The circumference as a geometric figure with infinite sides (dynamic conception) whose length approaches zero (metric conception).
   f. Curved shapes can be decomposed into square shapes (rectangles).
   g. If the sum of the areas of several subdivisions of a plane approaches infinity, then the margin of error of the total area approaches zero.
4. **Normative principle of social practice:** As the final stage of each activity, the institutionalization didactic situation is reached. Here, each mathematical object that constitutes the concept of the DI emerges from solutions to sociocultural problems implemented by the teacher. This is when the epistemological decomposition of the concept occurs, and the student builds another step in understanding the mathematical object of the DI, facilitating processes of interiorization–condensation–reification according to Sfard (1991).

Finally, these scopes described allow structuring a model for social appropriation of knowledge focused on social and historical practices derived from the construction, signification, and resignification of the concepts that make up knowledge. Due to the foregoing, under this ideal, the class scenarios in higher education are constituted as centers that permit the link among popular, technical, and scientific knowledge, in function of a use culturally situated at local, regional, national, and global levels.

Author contributions: CL-L: conception and design, data collection, analysis and interpretation, manuscript writing; **EA-B:** Conception and design, data collection, analysis and interpretation, manuscript writing, critical revision, funding; **A-MF-L:** conception and design, data collection, manuscript writing, critical revision. All authors agreed with the results and conclusions.

Funding: Eliécer Aldana Bermúdez received funding through the research committee of the University of Quindío.

Ethical statement: The authors stated that the research was conducted in accordance with ethical standards, ensuring the veracity of the data and respecting the privacy of the participants. The authors further stated that the study is the result of a Master's degree in Educational Sciences from the Universidad del Quindío. The authors undertake to provide access to the data and materials used to ensure the transparency of the study.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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